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K26P 1219

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree (C.B.C.S.S. – O.B.E. – Reg./Supple./Imp.) Examination, April 2026
(2023 Admissions Onwards)
MATHEMATICS/MATHEMATICS (Multivariate Calculus and Mathematics/ Analysis, Modelling and Simulation, Financial Risk Management)
MSMAT02C10/MSMAF02C10 : PDE and Integral Equations

Time : 3 Hours   Max. Marks : 80

PART – A

Answer **any 5** questions from this Part. **Each** question carries 4 marks. **(5×4=20)**

1. Define the Cauchy problem for a first-order quasilinear partial differential equation. Why is it called "quasilinear" ?
2. Classify the second order linear partial differential equations in two variables. Write their canonical forms with examples.
3. Explain the concept of domain of dependence of a point (x_0, t_0) and the region of influence of a point (x_0, t_0) for the Cauchy problem of the one dimensional homogeneous wave equation.
4. State the weak maximum principle.
5. State Gauss' theorem for a vector field ψ defined in a bounded piecewise smooth domain D and derive the first and second Green's identities from Gauss' theorem.
6. Show that the boundary value problem $y'' + \lambda y = 0$, $y(0) = 0$, $y(l) = 0$ satisfies a Fredholm equation of the second kind.

PART – B

Answer **any 3** questions from this Part. **Each** question carries **7** marks. **(3×7=21)**

7. Solve the Cauchy problem $u^2u_x + u_y = 0$, $u(x, 0) = \sqrt{x}$ in the ray $x > 0$. What is the domain of existence of the solution ?
8. Compute the function u(x, y) satisfying the eikonal equation $u_x^2 + u_y^2 = n^2$ and the initial condition $u(x,1) = n\sqrt{1+x^2}$, n is a constant parameter. Is the problem uniquely solvable ? Justify.

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9. State the Cauchy problem for the homogeneous wave equation and derive the corresponding d'Alembert's formula.
10. Let $D = \{(x, y) \in \mathbb{R}^2 | x^2 + y^2 > 1\}$ and $u(x, y) = \log(x^2 + y^2)$. Show that u is harmonic in D, $u(x, y) = 0$ on the boundary ∂D and $u(r, 0) \to \infty$ as $r \to \infty$ along the x-axis.
11. Solve the integral equation $y(x) = F(x) + \lambda\int_0^1 (1-3x\xi)y(\xi)d\xi$.

PART – C

Answer **any 3** questions from this Part. **Each** question carries **13** marks. **(3×13=39)**

12. a) Solve the Cauchy problem $xu_x - yu_y = u + xy$, $u(x, x) = x^2$, $1 \le x \le 2$.
    b) Draw the projections on the (x, y)-plane of the initial curve and the characteristic curves emanating from the points (1,1,1) and (2,2,4).
13. Consider the equation $u_{xx} - 2\sin x\, u_{xy} - \cos^2 x\, u_{yy} - \cos x\, u_y = 0$.
    a) Find a coordinate system $s = s(x, y)$, $t = t(x, y)$ in which the equation has its canonical form $v_{st} = 0$, and find the general solution.
    b) Find a solution of the equation which satisfies $u(0, y) = f(y)$, $u_x(0, y) = g(y)$, where f, g are given functions.
14. a) Show that the type of a linear second order PDE in two variables is invariant under a change of coordinates.
    b) Show that the Cauchy problem for the one-dimensional homogeneous wave equation is ill posed on the domain $-\infty < x < \infty$, $t \ge 0$.
15. a) State and prove the mean value principle for a harmonic function in a planar domain D.
    b) Show that the Dirichlet problem of vibrating string has a unique solution using energy method.
16. a) Solve using a method of successive approximations
    $y(x) = 1 + \lambda\int_0^1 (1-3x\xi)y(\xi)d\xi$.
    b) Let $y_m(x)$ and $y_n(x)$ be characteristic functions corresponding respectively in to two distinct characteristic numbers λ_m, λ_n on the homogeneous Fredholm equation $y(x) = \lambda\int_a^b K(x, \xi)y(\xi)d\xi$ and suppose that the kernel K is symmetric. Prove that y_m and y_n are orthogonal over (a, b).