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K26U 1302

Reg. No. : ....

Name : ....

IV Semester B.Sc. Honours in Mathematics Degree (CBCSS – OBE – Supplementary/Improvement) Examination, April 2026
(2021 to 2023 Admissions)
4B15 BMH : INTRODUCTION TO ABSTRACT ALGEBRA AND LINEAR ALGEBRA

Time : 3 Hours | Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Is subtraction a commutative binary operation on the set of integers ? Justify your answer.
2. Find the quotient and remainder when – 38 is divided by 7 according to division algorithm.
3. Find the order of (1, 4, 5, 7) in S_8.
4. Dimension of the vector space V = {0} is
5. Define kernel of linear transformation.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. Is the set of all n × n matrices with real entries under matrix multiplication a monoid ? Justify your answer.
7. State and prove left cancellation law in group.
8. Write the group table of Klein 4-group.
9. Define cycle and length of a cycle.

P.T.O.

10. Determine whether $\sigma : \mathbb{R} \to \mathbb{R}$ defined by $\sigma(x) = x + 1$ a permutation in $\mathbb{R}$.
11. Show that $\alpha 0 = 0$ for any $\alpha \in \mathbb{R}$.
12. Are the vectors $v_1 = (1, 2)^T$ and $v_2 = (1, -1)^T$ linearly independent ? Justify your answer.
13. Define linear transformation.
14. Write down the matrix of the linear transformation which change the basis in $\mathbb{R}^2$ by a rotation of the axes through an angle of π clockwise.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Prove that subgroup of a cyclic group is cyclic.
16. If G is a group with binary operation $*$, and if a and b are any elements of G, then prove that linear equations $a * x = b$ and $y * a = b$ have unique solutions x and y in G.
17. Find all proper subgroups of $\mathbb{Z}_{12}$.
18. Define orbit and find the orbit of the permutation $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 1 & 3 & 6 & 2 & 4 \end{pmatrix}$.
19. Consider the cycles $\sigma = (1, 4, 5, 6)$ and $\tau = (2, 1, 5)$ in S_6. Find $\sigma\tau$ and $\tau\sigma$.
20. Let G and G′ be groups and let $\varphi : G \to G'$ be a one-to-one function such that $\varphi(xy) = \varphi(x)\varphi(y)$ for all $x, y \in G$. Then prove that $\varphi[G]$ is a subgroup of G′ and φ provides an isomorphism of G with $\varphi[G]$.
21. Suppose V is a vector space. Then prove that for a non-empty subset W of V is a subspace if and only if for all $u, v \in W$, $u + v \in W$ (that is, W is closed under addition), and for all $v \in W$ and $\alpha \in \mathbb{R}$, $\alpha v \in W$ (that is, W is closed under scalar multiplication).
22. Suppose that a vector space V has a finite basis consisting of r vectors. Then prove that any basis of V consists of exactly r vectors.

23. Find the basis of the row space of the matrix $A = \begin{pmatrix} 1 & 2 & 1 & 3 & 0 \\ 0 & 1 & 1 & 1 & -1 \\ 1 & 3 & 2 & 0 & 1 \end{pmatrix}$.
24. Construct a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ with $N(T) = \left\{ t\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \middle| t \in \mathbb{R} \right\}$ and R(T) = xy-plane.
25. Find a basis of range space of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$.
26. Prove that the range and null space of a linear transformation $T : V \to W$ are subspaces of W and V, respectively.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. a) Define subgroup of a group.
    b) Prove that intersection of two subgroups of a group is again a subgroup of that group.
28. State and prove Cayley's Theorem.
29. Verify the rank-nullity theorem for the matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 0 & -1 & -1 \end{pmatrix}$.
30. State and prove rank-nullity theorem for linear transformations.

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