

Second Semester FYUGP Degree (Reg/Sup/Imp) Examination**April 2026****KU2DSCMAT117 - CALCULUS AND MATRIX ALGEBRA II**

2024 Admission onwards

Time : 2 hours

Maximum Marks : 70

Section A

Answer any 6 questions. Each carry 3 marks.

1. Write the n^{th} derivative of $a^{\tan x}$.
2. Write the n^{th} derivative of $\frac{1}{ax+b}$.
3. Write down the value of $\int_0^{\frac{\pi}{2}} \cos^7 x \, dx$.
4. Evaluate $\int_0^{\frac{\pi}{2}} \cos^5 x \sin^4 x \, dx$.
5. Evaluate $\int_0^{\frac{\pi}{2}} \cos^3 x \sin^4 x \, dx$.
6. Determine the inverse of the elementary matrix $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.
7. Define inverse of a matrix.
8. What is the relationship between eigenvalues of a matrix and those of its transpose?

Section B

Answer any 4 questions. Each carry 6 marks.

9. Evaluate $\int \sin^5 x \, dx$.
10. Evaluate $\int_0^1 \frac{x^5}{\sqrt{1-x^2}} dx$.
11. Evaluate $\int \sin^2 x \cos^6 x \, dx$.

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12. Find the inverse of the lower triangular matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ -1 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix}$.

13. Determine the eigenvalues and eigenvectors of $A = \begin{bmatrix} 3 & 4 \\ -5 & -5 \end{bmatrix}$.

14. Determine the eigenvalues of $A = \begin{bmatrix} 4 & 3 & 0 & 0 \\ -2 & -1 & 0 & 0 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & -3 & -2 \end{bmatrix}$.

Section C

Answer any 2 questions. Each carry 14 marks.

15. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x + 2y + z^2$, $x = \frac{r}{s}$, $y = r^2 + \ln s$, $z = 2r$.
16. Find the partial derivatives of the function $A(c, h, k, m, q) = \frac{km}{q} + cm + \frac{hq}{2}$ with respect to each variable.
17. (a) State Taylor's Theorem with Lagrange's form of remainder.
(b) If $f(x) = \log(1+x)$, $x > 0$, using Maclaurin's theorem, show that, for $0 < \theta < 1$,
 $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3(1+\theta x)^3}$. Deduce that $\log(1+x) < x - \frac{x^2}{2} + \frac{x^3}{3}$ for $x > 0$.