K26P 1218

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree (CBCSS – OBE – Reg./Supple./Imp.) Examination, April 2026
(2023 Admission Onwards)
MATHEMATICS/MATHEMATICS (Multivariate Calculus and Mathematical Analysis, Modelling and Simulation, Financial Risk Management)
MSMAT02C09/MSMAF02C09 : Advanced Topology

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any 5** questions from this Part. **Each** question carries **4** marks.

1. Define a well ordered set. Show that every well ordered set has least upper bound property.
2. Define a compact space. Show that a discrete space is compact if and only if it is finite.
3. Define first countable space. Show that every second countable space is first countable.
4. Prove that the Sorgenfrey line $\mathbb{R}_l$ is normal.
5. Show that a subspace of a completely regular space is completely regular.
6. Show that a connected normal space with more than one point is uncountable. **(5×4=20)**

PART – B

Answer **any 3** questions from this Part. **Each** question carries **7** marks.

7. Let $\mathcal{A}$ be an open covering of the metric space (X, d). If X is normal, show that there is a $\delta > 0$ such that for each subset of X having diameter less than δ, there exists an element of $\mathcal{A}$ containing it.

P.T.O.

8. Show that every sequentially compact metric space is compact.
9. Prove or disprove the following :
   a) The product of two Lindelöf spaces is Lindelöf.
   b) A subspace of a Lindelöf space is Lindelöf.
10. Show that a regular space with a countable basis is normal.
11. Let X be a completely regular space. If Y_1 and Y_2 are two compactifications of X satisfying the extension property, prove that Y_1 and Y_2 are equivalent. **(3×7=21)**

PART – C

Answer **any 3** questions from this Part. **Each** question carries **13** marks.

12. a) State and prove uniform continuity theorem.
    b) Show that limit point compactness does not imply compactness.
13. a) Prove that the space $\mathbb{R}_l$ satisfies all the countability axioms except the second.
    b) Show that the space $\mathbb{R}_K$ is not regular.
14. Show that every regular space with a countable basis is metrizable.
15. State and prove the Tietze Extension Theorem.
16. a) Show that the product of finitely many compact spaces is compact.
    b) Let X be a non-empty compact Hausdorff space. If X has no isolated points, show that X is uncountable. **(3×13=39)**

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