K26P 1217

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree (C.B.C.S.S. – O.B.E. – Reg./Supple./Imp.) Examination, April 2026

(2023 Admissions Onwards)

MATHEMATICS

MSMAT02C08 : Advanced Real Analysis

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any 5** questions from this Part. **Each** question carries **4** marks.

1. Define Uniform Continuity for a sequence of functions. Give an example with illustration.
2. Does every pointwise convergent sequence of functions contains a uniformly convergent subsequence ? Justify.
3. Let E be an open set in $\mathbb{R}^n$ and $f : E \to \mathbb{R}^m$ and $x \in E$. Define the derivative of f at x. If $f : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation, then prove that it is differentiable.
4. If K is a compact metric space, $f_n \in \mathcal{C}(K)$ for n = 1, 2, 3, ... and $\{f_n\}$ converges uniformly on K then prove that $\{f_n\}$ is equicontinuous on K.
5. If for some x, there are constants $\delta > 0$ and $M < \infty$ such that $|f(x + t) - f(x)| \le M|t|$ for all $t \in (-\delta, \delta)$ then prove that $\lim_{N\to\infty} S_N(f; x) = f(x)$.
6. State implicit function theorem. **(5×4=20)**

PART – B

Answer **any 3** questions from this Part. **Each** question carries **7** marks.

7. If $\{f_n\} \to f$ uniformly on a set E in a metric space and x is a limit point of E with $\lim_{t\to x} f_n(t) = A_n$ for n = 1, 2, 3, ... then prove that $\{A_n\}$ converges and $\lim_{t\to x} f_n(t) = \lim_{n\to\infty} A_n$.

P.T.O.

8. Suppose $\{f_n\}$ is a sequence of functions, differentiable on [a, b] and such that $\{f_n(x_0)\}$ converges for some x_0 on [a, b]. If $\{f_n'\}$ converges uniformly on [a, b], then show that $\{f_n\}$ converges uniformly on [a, b], to a function f, and $f'(x) = \lim_{n\to\infty} f_n'(x)$ $(a \le x \le b)$.
9. Let $\{a_{ij}\}$ for i, j = 1, 2, 3, ... and $\sum_{j=1}^{\infty} |a_{ij}| = b_i$ for i = 1, 2, 3, ... and $\sum b_i$ converges, then show that $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_{ij} = \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} a_{ij}$.
10. If f is a positive function on $(0, \infty)$ such that $f(x + 1) = f(x)$, $f(1) = 1$, log f is convex then prove that $f(x) = \Gamma(x)$.
11. a) Define a contraction mapping.
    b) If X is a complete metric space and $\phi : X \to X$ is a contraction mapping then prove that there exists a unique point $x \in X$ such that $\phi(x) = x$. **(3×7=21)**

PART – C

Answer **any 3** questions from this Part. **Each** question carries **13** marks.

12. Prove that the collection $\mathcal{C}(X)$, the set of all complex valued, continuous, bounded functions with domain a metric space X is a complete metric space.
13. a) Define an algebra.
    b) Let $\mathcal{A}$ be an algebra of real continuous functions on a compact set K. If $\mathcal{A}$ separates points on K and if $\mathcal{A}$ vanishes at no point of K, then prove that the uniform closure $\mathcal{B}$ of $\mathcal{A}$ consists of all real continuous functions on K.
14. Prove that there exists a real continuous function on the real line which is nowhere differentiable.
15. If $\sum c_n$ converges and $f(x) = \sum_{n=0}^{\infty} c_n x^n$ for $|x| < 1$ then show that $\lim_{x\to 1} f(x) = \sum_{n=0}^{\infty} c_n$.
16. State and prove inverse function theorem. **(3×13=39)**

____