K26FY1216 Pages: 2 Reg No:..................
Name:..................

Second Semester FYUGP Degree (Reg/Sup/Imp) Examination April 2026
KU2DSCMAT101 - CALCULUS II
2024 Admission onwards

Time : 2 hours Maximum Marks : 70

Section A

Answer any 6 questions. Each carry 3 marks.

1. Evaluate $\int_0^{\pi} \sin^5\left(\frac{x}{2}\right)\, dx$.
2. Evaluate $\int \sin^5 x\, dx$.
3. Determine the 20^{th} derivative of e^{2x}.
4. Verify whether Rolle's theorem is applicable for the function $f(x) = x^2 - 4x + 3$ on $[1, 3]$.
5. Does the function $f(x) = |x|$ satisfy Rolle's theorem on $[-1, 1]$. Justify your answer.
6. Determine the limit $\lim_{x\to\frac{\pi}{2}^-} \frac{\sec x}{1+\tan x}$.
7. Find the intervals on which the function $f(x) = -x^3 + 2x^2$ is decreasing.
8. Calculate $\lim_{x\to\infty} x^2 e^{-x}$.

Section B

Answer any 4 questions. Each carry 6 marks.

9. If $y = \frac{ax+b}{cx+d}$, show that $2y_1y_3 = 3y_2^2$.
10. If $x = \sin t$ and $y = \sin pt$, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$.
11. Prove that if two differentiable functions $f(x)$ and $g(x)$ satisfy $f'(x) = g'(x)$ for all x in an open interval (a, b), then they differ by a constant.
12. Describe the first derivative test for local extreme values of a continuous function f. Identify the local and absolute extreme values of the function $f(x) = (x+1)^2$, $-\infty < x \le 0$.

13. An open-top box is to be made by cutting small congruent squares from the corners of a 12-in-by-12-in. sheet of tin and bending up the sides. How large should the squares cut from the corners be to make the box hold as much as possible?
14. Evaluate $\lim_{x\to 0^+} \left(1 + \frac{1}{x}\right)^x$.

Section C

Answer any 2 questions. Each carry 14 marks.

15. Find and sketch the domain for the function $f(x, y) = \frac{1}{\ln(4 - x^2 - y^2)}$. Find the range of this function.
16. (a) Let $f(x, y) = \begin{cases} x^2, & x \ge 0 \\ x^3, & x < 0 \end{cases}$.

    Compute the following limits.

    (i) $\lim_{(x,y)\to(3,-2)} f(x, y)$

    (ii) $\lim_{(x,y)\to(-2,1)} f(x, y)$

    (iii) $\lim_{(x,y)\to(0,0)} f(x, y)$.

    (b) Define $f(0, 0)$ in a way that extends $f(x, y) = \ln\left(\frac{3x^2 - x^2y^2 + 3y^2}{x^2+y^2}\right)$ to be continuous at the origin
17. Write the reduction formula for $\int \sin^p x \cos^q x\, dx$ and hence evaluate the definite integral $\int_0^{\pi/2} \sin^p x \cos^q x\, dx$, where p and q are positive integers.