K26U 0902

Reg. No. : ....

Name : ....

Second Semester B.Sc. (Hons) Mathematics Degree (CBCSS – OBE – Supplementary) Examination, April 2026
(2021 – 2023 Admissions)
CORE COURSE
2B07 BMH : Theory of Numbers and Equations

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any four** questions out of five questions. **Each** question carries **1** mark.

1. State Bonse's inequality.
2. State True/False : $7 \equiv 3 \pmod 5$.
3. Find $\tau(12)$.
4. Find all roots of $f(x) = x^2 + x - 6$.
5. State Descarte's rule for positve roots. **(4x1=4)**

SECTION – B

Answer **any six** questions out of nine questions. **Each** question carries **2** marks.

6. Prove that square of any integer is either 3k or 3k + 1.
7. For integers a and b, prove the following :
   If a|b and $b \neq 0$, then $|a| \leq |b|$.
8. Find the solution, if there is any, of $2x + 8y = 13$.
9. Solve the following epigram problem : Diophantus's boyhood lasted $\frac{1}{6}$ of his life; his beard grew after $\frac{1}{12}$ more; after $\frac{1}{7}$ more he married and his son was born 5 years later; the son lived to half his father's age and the father died 4 years after his son. Then at what age Diophantus died ?

P.T.O.

10. Define Euclidean numbers.
11. Let n > 1 be fixed and a, b be arbitrary integers. Then prove that $a \equiv b \pmod n$ implies $b \equiv a \pmod n$.
12. Show that $n^7 - n$ is divisible by 42.
13. Solve the system of equations :
    $x + ay + a^2z = a^3$, $x + by + b^2z = b^3$, $x + cy + c^2z = c^3$.
14. Factorize $(x + y + z)^5 - x^5 - y^5 - z^5$. **(6x2=12)**

SECTION – C

Answer **any eight** questions out of twelve questions. **Each** question carries **4** marks.

15. Let a and b be integers, not both zero. Then prove that a and b are relatively prime if and only if there exist integers x and y such that 1 = ax + by.
16. Let a, b be integers, not both zero. Prove that if d = gcd (a, b) then the following are satisfied.
    a) d|a and d|b
    b) Whenever c|a and c|b, then c|d.
17. A customer bought a dozen pieces of fruit, apples and oranges, for Rs. 132. If an apple costs 3 rupees more than an orange and more apples than oranges were purchased, how many pieces of each kind were bought ?
18. Prove that if the linear Diophantine equation ax + by = c has a solution then d|c, where d = gcd(a, b).
19. Prove that there is an infinite number of primes.
20. Using Sieve of Eratosthenes find all primes not exceeding 140.
21. Find the remainder obtained upon dividing the sum
    $1! + 2! + 3! + 4! + ... + 99! + 100!$ by 12.
22. State and prove Chinese Remainder Theorem.
23. Solve the linear congruence $9x \equiv 21 \pmod{30}$.
24. If p is a prime number, then prove that $x^p - x$ is divisible by p.
25. Solve $6x^5 + 11x^4 - 33x^2 + 11x + 6 = 0$.
26. Determine the nature of the roots of the equation $7x^5 - 5x^2 - 4x + 5 = 0$. **(8x4=32)**

SECTION – D

Answer **any two** questions out of four questions. **Each** question carries **6** marks.

27. Given integers a and b, with b > 0, then prove that there exist unique integers q and r satisfying $a = qb + r$, $0 \leq r < b$.
28. Prove that if the quadratic congruence $x^2 + 1 \equiv 0 \pmod p$, where p is an odd prime, has a solution then $p \equiv 1 \pmod 4$.
29. Prove that the functions τ and σ are both multiplicative functions.
30. Using Cardon's method, solve the equation $x^3 - 6x - 9 = 0$. **(2x6=12)**