**K26U 0865**

Reg. No. : ..............................

Name : ..................................

**Second Semester B.Sc. Degree (C.B.C.S.S.-OBE – Supplementary) Examination, April 2026**
**(2020 to 2023 Admissions)**
**CORE COURSE IN STATISTICS**
**2B02STA : Probability Theory and Mathematical Expectation**

Time : 3 Hours | Max. Marks : 48

PART – A

Short Answer. Answer **all** questions. **1** mark **each**.

1. Give an example for a random experiment.
2. What do you meant by exhaustive events ?
3. Define probability density function.
4. Define distribution function of a bivariate random variable.
5. If the first two raw moments about origin of a random variable are 5 and 41 respectively, obtain the standard deviation.
6. If $X_1$ and $X_2$ are independent random variables with mean 5 and 2 respectively find the mean of $Y = X_1 X_2$. **(6×1=6)**

PART – B

Short Essay. Answer **any 7** questions. **2** marks **each**.

7. Discuss the frequency approach to probability.
8. If A and B are independent events such that $P(A) = 0.6$ and $P(A \cup B) = 0.8$. Find P(B).
9. Check whether the function defined by $f(x) = \frac{3+2x}{18}$, $2 < x < 4$ is a pdf ?

P.T.O.



10. Define probability mass function of a discrete random variable and discuss its properties.
11. Show that E(c) = c, where c is a constant.
12. If X is a random variable with pdf $f(x) = e^{-x}$, $0 < x < \infty$, find $E(X^2 + 1)$.
13. If $E(X - 4) = -2$, $E(X - 4)^2 = 12$, find the mean and variance of the random variable X.
14. Define conditional expectation and state its properties.
15. Let X be a random variable with pgf P(s). Find the pgf of $Y = X + 1$. **(7×2=14)**

PART – C

Essay answer **any 4** questions. **4** marks **each**.

16. If A is a subset of B, prove that (i) $P(\bar{A} \cap B) = P(B) - P(A)$ (ii) $P(A) \le P(B)$.
17. If A and B are independent events, show that $\bar{A}$ and $\bar{B}$ are also independent.
18. Let X be a discrete random variable assuming values $-2, -1, 0, 1, 2$ such that $P(X = -2) = P(X = -1)$, $P(X = 1) = P(X = 2)$ and $P(X > 0) = P(X < 0) = P(X = 0)$. Obtain the pmf and distribution function of X.
19. Let the random variables X and Y are such that $P(X = 0, Y = 0) = \frac{2}{9}$, $P(X = 0, Y = 1) = \frac{1}{9}$, $P(X = 1, Y = 0) = \frac{1}{9}$, $P(X = 1, Y = 1) = \frac{5}{9}$. Examine whether X and Y are independent random variables.
20. Establish the relation between raw moments and central moments of a random variable.
21. Define cumulant generating function of a random variable. State and prove the additive property of cumulant generating function. **(4×4=16)**



PART – D

Essay answer **any 2** questions. **6** marks **each**.

22. i) State Bayes' theorem.

    ii) It is estimated that 50% of emails are spam emails. Some software has been applied to filter these spam emails before they reach your inbox. A certain brand of software claims that it can detect 99% of spam emails and the probability for a false positive (a non-spam email detected as spam) is 5%. Now if an email is detected as spam, then what is the probability that it is in fact a non-spam email ?

23. A discrete random variable X has distribution function

$$F(x) = \begin{cases} k_1, & -\infty < x < -1 \\ k_2, & -1 \le x < 0 \\ k_3, & 0 \le x < 2 \\ k_4, & 2 \le x < \infty \end{cases}$$

Also given that $P(X = 0) = \frac{1}{6}$ and $P(X > 1) = \frac{2}{3}$. Determine the values of $k_1$, $k_2$, $k_3$, and $k_4$.

24. If X and Y are any two random variables, then prove the following. (i) $V(aX + b) = a^2 V(X)$ (ii) $Cov(aX, bY) = ab\, Cov(X, Y)$
25. Define characteristic function of a random variable. If the random variable is symmetric about 0, then show that the characteristic function $\varphi(t)$ is real valued and even function of t. **(2×6=12)**

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