Reg. No. : ....

Name : ....

IV Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – O.B.E. – Supplementary/Improvement) Examination, April 2026
(2021 to 2023 Admissions)
4B14 BMH : ADVANCED REAL ANALYSIS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Give an example of unbounded function.
2. Define Riemann integrable function.
3. Give an example of sequence of function $f_n(x)$ on [0, 1] such that $\lim \int_0^1 f_n(x) \neq \int_0^1 \lim f_n(x)$.
4. Define gamma function.
5. Write the relation between beta and gamma functions.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. Show that the equation x = cos x has a solution in the interval $\left[0, \frac{\pi}{2}\right]$.
7. State Nonuniform Continuity Criteria.
8. Find the absolute maximum and absolute minimum of the function $f(x) = x^2$ on [–1, 1].
9. Is it possible to apply "Fundamental Theorem of Calculus First Form" to the function $H(x) = 2\sqrt{x}$ for $x \in [0, b]$? Justify your answer.
10. If $\varphi : [a, b] \to \mathbb{R}$ is a step function then prove that $\varphi \in \mathcal{R}[a, b]$.

P.T.O.

11. Discuss the uniform convergence of the series $\sum \frac{1}{x^2 + n^2}$.
12. State Cauchy criterion for uniform convergence of series of functions.
13. "If the Cauchy Principal Value of $\int_{-\infty}^{\infty} f(x)\,dx$ exists then $\int_{-\infty}^{\infty} f(x)\,dx$ converges". Is this statement true ? Justify your answer.
14. Evaluate $\int_0^{\infty} e^{-x^2} dx$.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. State and prove Bolzano's Intermediate Value Theorem.
16. If $f : A \to \mathbb{R}$ is uniformly continuous on a subset A of $\mathbb{R}$ and if (x_n) is a Cauchy sequence in A, then prove that $(f(x_n))$ is a Cauchy sequence in $\mathbb{R}$.
17. Is every uniformly continuous function a Lipschitz function ? Justify your answer.
18. If $f : [a, b] \to \mathbb{R}$ is monotone on [a, b] then prove that $f \in \mathcal{R}[a, b]$.
19. Let $g : [0, 3] \to \mathbb{R}$ be defined by g(x) = 2 for $0 \le x \le 1$ and g(x) = 3 for $1 < x \le 3$. Then prove that $g \in \mathcal{R}[0, 3]$ and $\int_0^3 g = 8$.
20. Let f and g are in $\mathcal{R}[a, b]$ then prove that f + g is in $\mathcal{R}[a, b]$. Also prove that if $f(x) \le g(x)$ for all x in [a, b] then $\int_a^b f \le \int_a^b g$.
21. Let $G_n(x) = x^n(1 - x)$ for $x \in [0, 1]$. Does $(G_n(x))$ converge uniformly on [0, 1] ? Justify your answer.
22. Prove that a sequence (f_n) of bounded functions on $A \subseteq \mathbb{R}$ converges uniformly on A to f if and only if $\|f_n - f\|_A \to 0$.
23. Define radius of convergence of a power series and find the radius of convergence of the power series $\sum \frac{1}{n^n} x^n$.

24. Check whether the improper integral $\int_0^1 \frac{\sin x}{\sqrt{x}} dx$ converges absolutely.
25. Express $\int_0^1 x^m (1 - x^n)^p dx$ in terms of Gamma functions.
26. Prove that $\int_0^{\frac{\pi}{2}} \frac{\cos^{2m-1}\theta \sin^{2n-1}\theta\, d\theta}{a\cos^2\theta + b\sin^2\theta} = \frac{\beta(m, n)}{2a^m b^n}$.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. State and prove the Continuous Extension Theorem.
28. Prove that Thomae's function is Riemann integrable.
29. State and prove Weierstrass M-Test and use it to show that the series $\sum_{n=1}^{\infty} \frac{1}{n^3 + x^2 n^4}$ is uniformly convergent for all values of x.
30. State and prove properties of beta function.