

Fourth Semester FYUGP Degree (Reg) Examination April 2026

KU4DSCCMT203 - MULTIVARIABLE CALCULUS

2024 Admission onwards

Time : 2 hours

Maximum Marks : 70

Section A

Answer any 6 questions. Each carry 3 marks.

1. Evaluate $\lim_{(x,y) \rightarrow (5,-2)} x^2y + 3xy^2 + 4$
2. Evaluate $\lim_{(x,y) \rightarrow (1,1)} \frac{x^2y^3 + x^3y^2}{x^2 + y^2}$
3. write the formula to find area of the surface given by $z = f(x, y)$
4. The point $(0, 2\sqrt{3}, -2)$ is given in rectangular coordinates. Find spherical coordinates for this point
5. Define triple integral of a function over a box B
6. Explain why the function $f(x, y) = x^3y^2$ is differentiable at the point $(-2, 1)$
7. If $x^3 + y^3 = 6xy$, find $\frac{dy}{dx}$
8. Find the directional derivative $D_{\mathbf{u}}f(x, y)$ if $f(x, y) = x^3 - 3xy + 4y^2$ and $\mathbf{u}$ is the unit vector in the direction given by angle $\theta = \pi/6$

Section B

Answer any 4 questions. Each carry 6 marks.

9. Find the area of the part of the paraboloid $z = x^2 + y^2$ that lies under the plane $z = 9$. the rectangle $[1, 4] \times [2, 6]$
10. Find the surface area of the part of the surface $z = x^2 + 2y + 2$ that lies above the triangular region T in the XY-plane with vertices $(0, 0)$, $(1, 0)$, and $(1, 1)$.
11. Evaluate $\int \int \int_B e^{(x^2+y^2+z^2)^{3/2}} dv$, where B is the unit ball $B = \{(x, y, z) \mid x^2 + y^2 + z^2 \leq 16\}$.
12. (a) If $z = f(x, y) = x^2 + 3xy + y^2$, find the differential dz .
(b) If x changes from 2 to 2.05 and y changes from 3 to 2.96 compare the values of Δz and dz .
13. $f(x, y) = xe^y$

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- (a) Find the rate of change of f at the point P $(2, 0)$ in the direction from P to Q $(1/2, 2)$
- (b) In what direction does f have the maximum rate of change? What is this maximum rate of change?

14. Find the extreme values of $f(x, y) = y^2 - x^2$.

Section C

Answer any 2 questions. Each carry 14 marks.

15. Find the mass and center of mass of a triangular lamina with vertices $(0, 0)$, $(1, 0)$, and $(0, 2)$ if the density function is $\rho(x, y) = 1 + 3x + y$.
16. Find the extreme values of $f(x, y) = -x^2 + 2y^2$ on the disk $D = \{(x, y) \mid x^2 + y^2 \leq 1\}$ using Lagrange multiplier Method
17. Show that the following limit does not exist
 - (a) $\lim_{(x,y) \rightarrow (0,0)} \frac{y^2}{x^2+y^2}$
 - (b) $\lim_{(x,y) \rightarrow (0,0)} \frac{2xy}{x^2+3y^2}$
 - (c) $\lim_{(x,y) \rightarrow (0,0)} \frac{(x+y)^2}{x^2+y^2}$
 - (d) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2+xy^2}{x^4+y^2}$