Reg. No. : ....

Name : ....

IV Semester M.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supple./Imp.) Examination, April 2026
(2023 Admission Onwards)
MATHEMATICS
MSMAT04C15 : Complex Function Theory

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any five** questions. **Each** question carries **4** marks. **(5×4=20)**

1. Show that $\prod_{n=1}^{\infty}\left(\frac{1}{2}\right)$ is convergent.
2. Show that $\sum_{n=1}^{\infty} n^{-z}$ converges uniformly and absolutely on $\{z : \text{Re } z \geq 1 + \in\}$.
3. Let G be the region that is symmetric with respect to the real axis and $f : G \to \mathbb{C}$ be analytic. Show that $f(z) = \overline{f(\overline{z})}$, for all $z \in G$, if f(x) is real for all $x \in G \cap \mathbb{R}$.
4. State Monodromy theorem.
5. Write Harnack's inequality.
6. Define a harmonic function and give one example.

PART – B

Answer **any three** questions. **Each** question carries **7** marks. **(3×7=21)**

7. a) Use definition of convergence of an infinite product to compute $\prod_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right)$.

   b) Prove or disprove : an absolutely convergent infinite product is convergent.
8. State and prove Euler's theorem for ζ-function.
9. Explain the terms function element and germ with example.

10. If $u : G \to \mathbb{R}$ is a continuous function which has the MVP, show that u is harmonic.
11. Let G be a bounded region and suppose that $w : G^{-} \to \mathbb{R}$ is a continuous function that satisfies the MVP on G. If w(z) = 0 for all z in ∂G, then show that w(z) = 0 for all z in G.

PART – C

Answer **any three** questions. **Each** question carries **13** marks. **(3×13=39)**

12. a) Let $\{a_n\}$ be a sequence in $\mathbb{C}$ such that $\lim |a_n| = \infty$ and $a_n \neq 0$ for all $n \geq 1$. Show that $f(z) = \prod_{n=1}^{\infty} E_{p_n}(z/a_n)$ converges in $H(\mathbb{C})$, if $\sum_{n=1}^{\infty}\left(\frac{r}{|a_n|}\right)^{p_n+1} < \infty$ for all $r > 0$, where $\{p_n\}$ is any sequence of integers.

    b) Use the above result to prove Weierstrass function theorem.
13. Derive Riemann's functional equation.
14. State and prove Schwarz reflection principle.
15. a) Define Poisson Kernel.

    b) Show that the Poisson Kernel satisfies the following :

    i) $\frac{1}{2\pi}\int_{-\pi}^{\pi} P_r(\theta)\,d\theta = 1$

    ii) $P_r(\theta) > 0$ for all θ, $P_r(-\theta) = P_r(\theta)$, P_r is periodic in θ with period 2π.
16. State and prove Harnack's theorem.