K26FY0319 Pages: 2 Reg No:.................

Name:.................

Fourth Semester FYUGP Degree (Reg) Examination April 2026

KU4DSCCMT202 - GROUP THEORY

2024 Admission onwards

Time : 2 hours Maximum Marks : 70

Section A

Answer any 6 questions. Each carry 3 marks.

1. Show that a group with only one element is abelian.
2. What is a structural property?
3. Give an example for a binary operation on $\mathbb{N}$.
4. Define the inverse of an element in a group.
5. Give one example of an infinite non-abelian group.
6. When are two cycles said to be disjoint?
7. What is the length of the cycle $(2\ 5\ 7\ 1)$?
8. What is the order of A_n?

Section B

Answer any 4 questions. Each carry 6 marks.

9. Construct a commutative group table for 4 element set.
10. Consider the set $S = \{0, 1, 2\}$ with operation $*$ given by:

$*$	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

a) Verify closure and associativity.
b) Identify the identity element.
c) Find inverses of all elements.

11. Let G be a group. Prove that for all $a, b \in G$,

$$(a * b)' = b' * a'.$$

12. Express the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 2 & 6 & 3 & 7 & 4 & 5 & 1 \end{pmatrix}$$

as a product of disjoint cycles and then as a product of transpositions.

13. Show that product of two even permutations is even.
14. Prove that every permutation can be written as a product of transpositions.

Section C

Answer any 2 questions. Each carry 14 marks.

15. Define a permutation group. Show that the set of all permutations of a finite set forms a group under composition.
16. a) Define a cyclic group. Prove that every cyclic group is abelian.
b) Give examples of finite and infinite cyclic groups.
17. Let G be a group and $a \in G$. Prove that

$$\langle a \rangle = \{a^n \mid n \in \mathbb{Z}\}$$

is a subgroup of G. Also show that it is the smallest subgroup containing a.