K26FY0259 Pages: 2 Reg No:.................

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# Fourth Semester FYUGP Degree (Reg) Examination April 2026

# KU4DSCSTA203 - BIVARIATE RANDOM VARIABLES

2024 Admission onwards

Time : 1.5 hours Maximum Marks : 50

## Section A

**Answer any 6 questions. Each carry 2 marks.**

1. If the cumulative distribution function of a continuous random variable $X$ is $F(x)$, find the cumulative distribution function of $Y = X + a$.
2. Define univariate transformation of random variables.
3. If $X$ and $Y$ are independent, write the relationship between their joint mgf and marginal mgfs.
4. Show how to obtain $E(XY)$ from the bivariate mgf.
5. Distinguish between convergence in probability and convergence in distribution.
6. What is the importance of the Central Limit Theorem in statistics?
7. Write any two properties of joint distribution function.
8. How is the conditional distribution obtained from a joint distribution?

## Section B

**Answer any 4 questions. Each carry 6 marks.**

9. If $X_n \xrightarrow{P} X$, prove that for any real number $a$,

$$aX_n \xrightarrow{P} aX.$$

10. State and prove Markov's inequality .

11. Examine whether the weak law of large numbers holds for the sequence of independent random variables $\{X_i\}$, $i = 1, 2, 3, \ldots$ where

$$P\left(X_i = \pm\sqrt{2i-1}\right) = \frac{1}{2}.$$



12. If

$$f(x,y) = k, \quad 0 < x < 1,\ 0 < y < 1.$$

is the joint probability density function of $(X, Y)$,

(i) Find $k$.
(ii) Find $P(X > 0.4, Y > 0.6)$.
(iii) Find the joint distribution function $F(x, y)$.

13. Given the joint probability masses of $(X, Y)$ as

$$f(x,y) = \frac{1}{4}, \quad \text{for } (x,y) = (1,1), (1,2), (2,1), (2,2).$$

Examine whether $X$ and $Y$ are independent.

14. Given that the joint probability density function is

$$f(x,y) = k(2x+3)e^{-y/2}, \quad 0 < x < 2,\ y > 0.$$

find:

(a) the value of $k$,
(b) show that $X$ and $Y$ are independent.

## Section C

**Answer any 1 questions. Each carry 14 marks.**

15. Let the joint pdf of the random variables $(X, Y)$ be

$$f(x,y) = \begin{cases} \alpha^{-2} e^{-\frac{x-y}{\alpha}}, & x > 0,\ y > 0,\ \alpha > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Find the distribution of $\frac{1}{2}(X - Y)$.

16. Let $X$ and $Y$ are two random variables with joint pmf

$$f(x,y) = \frac{x+2y}{18}, \quad x = 1, 2;\ y = 1, 2.$$

Find correlation between $X$ and $Y$ .