K25U 2931

Reg. No. : ....

Name : ....

III Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Supplementary/ Improvement) Examination, November 2025

(2019 to 2023 Admissions)

CORE COURSE IN MATHEMATICS

3B03 MAT : Analytic Geometry and Applications of Derivatives

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any 4** questions. **Each** question carries **1** mark. **(4×1=4)**

1. Give polar equation of a conic with eccentricity e.
2. Write equation of the normal at the point (x, y) of the curve y = f(x).
3. Find the length of the tangent of the curve $x = a\left(\cos t - \log\tan\frac{t}{2}\right)$, $y = a\sin t$.
4. State Lagrange's mean value theorem.
5. Define point of inflection of a function.

PART – B

Answer **any 8** questions. **Each** question carries **2** marks. **(8×2=16)**

6. Find foci and vertices of the ellipse $\frac{(x+3)^2}{9} + \frac{(y+2)^2}{25} = 1$.
7. Find the directrix of the parabola $r = \frac{6}{2 + \cos\theta}$.
8. Find the equation of the tangent to the curve y (x – 2) (x – 3) – x + 7 = 0 at the point where it cuts the x-axis.
9. For the cardioid r = a (1 – cos θ), prove that $p = 2a\sin^3\left(\frac{\theta}{2}\right)$.
10. Find the angle of intersection of the curves r = 2 sin θ and r = 2 cos θ.

P.T.O.

11. Show that the radius of curvature at any point of the cardioid $r = a(1-\cos\theta)$ is $\frac{2\sqrt{2ar}}{3}$.

12. Find the asymptotes of the curve $r = a(\sec\theta + \tan\theta)$.

13. State Taylor's theorem.

14. Using Maclaurin's series, expand e^x.

15. Determine the concavity of $y = 3 + \sin x$ on $[0, 2\pi]$.

16. Find $\lim\limits_{x \to 0} \frac{x - \sin x}{x^3}$.

PART – C

Answer **any 4** questions. **Each** question carries **4** marks. **(4×4=16)**

17. Find a Cartesian equation for the hyperbola centered at the origin that has focus at (3, 0) and the line x = 1 as the corresponding directrix.

18. Find the eccentricity of the ellipse $169\,x^2 + 25\,y^2 = 4225$. Also find and graph the ellipse's foci and directrices.

19. Show that the parabolas $y^2 = 4ax$ and $2x^2 = ay$ interest at angle $\tan^{-1}\left(\frac{3}{5}\right)$.

20. Prove that, in parabola $\frac{2a}{r} = 1 - \cos\theta$,

    i) $\phi = \pi - \frac{\theta}{2}$ and

    ii) $\pi = a \operatorname{cosec}\left(\frac{\theta}{2}\right)$.

21. Find the radius of curvature at any point $(at^2, 2at)$ of the parabola $y^2 = 4ax$.

22. Verify Cauchy's mean value theorem for the functions e^x and e^{-x} in the interval (a, b).

23. Find the following limits

   i) $\lim\limits_{x \to \infty} \frac{\ln x}{2\sqrt{x}}$

   ii) $\lim\limits_{x \to \infty} x\left(\sin\frac{1}{x}\right)$.

PART – D

Answer **any 2** questions. **Each** question carries **6** marks **(2×6=12)**

24. The hyperbola $\frac{y^2}{4} - \frac{x^2}{5} = 1$ is shifted 2 units down.

   i) Find the equation of the new hyperbola in standard form.

   ii) Find the center, foci, vertices and asymptotes of the new hyperbola.

   iii) Plot the new hyperbola.

25. Show that the condition for the curves $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ should intersect orthogonally is that $\frac{1}{a} - \frac{1}{b} = \frac{1}{a'} - \frac{1}{b'}$.

26. Show that the evolute of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ is another equal cycloid.

27. If $f(x) = \log(1 + x)$; $x > 0$, using Maclaurin's theorem, show that for $0 < \theta < 1$ $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3(1+\theta x)^3}$. Deduce that $\log(1+x) < x - \frac{x^2}{2} + \frac{x^3}{3}$ for $x > 0$.