



K25U 3083

Reg. No. :

Name :

**III Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – O.B.E. –
Supplementary/Improvement) Examination, November 2025
(2021 to 2023 Admissions)
3B11 BMH : GRAPH THEORY**

Time : 3 Hours

Max. Marks : 60

PART – A

Answer **any four** questions from this Part. **Each** question carries 1 mark. **(4×1=4)**

1. Define self-complementary graph.
2. Draw the Peterson graph.
3. For a connected graph G has 21 vertices, what is the minimum possible number of edges in G ?
4. Which of the wheel graphs W_n , for $n \geq 4$ are Euler ?
5. Define k -critical graphs.

PART – B

Answer **any six** questions from this Part. **Each** question carries 2 marks. **(6×2=12)**

6. State and prove first theorem of graph theory.
7. Define self-complementary graph. Give an example.
8. Prove that there is no simple graph on four vertices, three of which have degree 3 and the remaining vertex has degree one.
9. State two equivalent conditions of trees.
10. Let G be a graph with exactly one spanning tree. Prove that G is a tree.
11. State travelling salesman problem.

P.T.O.

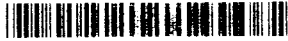


12. Prove that a tree has at most one perfect matching.
13. Let G be a k -critical graph. Then prove that the degree of every vertex of G is at least $k - 1$.
14. Prove that any sub-graph of a planar graph is planar.

PART – C

Answer **any 8** questions from this Part. **Each** question carries **4** marks. (8×4=32)

15. Prove that if u is an odd vertex in a connected graph G then there must be a path in G from u to another odd vertex v of G .
16. Let G be a k -regular graph, where k is an odd number. Prove that the number of edges in G is a multiple of k .
17. Given any two vertices u and v of a graph G , every $u - v$ walk contains a $u - v$ path.
18. If T is a tree with n vertices then prove that it has precisely $n - 1$ vertices.
19. Let G be a connected graph. Then G is a tree if and only if every edge of G is a bridge.
20. Let v be a vertex of the connected graph G . Then prove that v is a cut vertex of G if and only if there are two vertices u and w of G , both different from v , such that v is on every $u - w$ path in G .
21. Explain Konigsberg Bridge problem and its graph model.
22. Prove that a simple graph G is Hamiltonian if and only if its closure $c(G)$ is Hamiltonian.
23. Define Hamiltonian graphs. Let G be a simple graph with n vertices and let u and v be non-adjacent vertices in G such that $d(u) + d(v) \geq n$. Then prove that G is Hamiltonian if and only if $G + uv$ is Hamiltonian.
24. Prove that a graph G is planar if and only if it has no sub-graph isomorphic to a sub-division of K_5 or $K_{3,3}$.



25. Let G be a graph. Then prove that $\chi(G) \geq 3$ if and only if G has an odd cycle.
26. Let G be a nonempty graph. Then prove that $\chi(G) = 2$ if and only if G is bipartite.

PART – D

Answer **any 2** questions from this Part. **Each** question carries **6** marks. (2×6=12)

27. Explain the Matrix representation of graphs with example.
28. Prove that a graph G is connected if and only if it has a spanning tree.
29. Let G be a graph with n vertices and q edges. Then prove that G has at least $n - \omega(G)$ edges.
30. State and prove Euler's formula for planar graphs.
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