Reg. No. : ....

Name : ....

K25U 3083

III Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – O.B.E. – Supplementary/Improvement) Examination, November 2025
(2021 to 2023 Admissions)
3B11 BMH : GRAPH THEORY

Time : 3 Hours

Max. Marks : 60

PART – A

Answer **any four** questions from this Part. **Each** question carries **1** mark. **(4×1=4)**

1. Define self-complementary graph.

2. Draw the Peterson graph.

3. For a connected graph G has 21 vertices, what is the minimum possible number of edges in G ?

4. Which of the wheel graphs W_n, for $n \geq 4$ are Euler ?

5. Define k-critical graphs.

PART – B

Answer **any six** questions from this Part. **Each** question carries **2** marks. **(6×2=12)**

6. State and prove first theorem of graph theory.

7. Define self-complementary graph. Give an example.

8. Prove that there is no simple graph on four vertices, three of which have degree 3 and the remaining vertex has degree one.

9. State two equivalent conditions of trees.

10. Let G be a graph with exactly one spanning tree. Prove that G is a tree.

11. State travelling salesman problem.

P.T.O.

12. Prove that a tree has at most one perfect matching.

13. Let G be a k-critical graph. Then prove that the degree of every vertex of G is at least k – 1.

14. Prove that any sub-graph of a planar graph is planar.

PART – C

Answer **any 8** questions from this Part. **Each** question carries **4** marks. **(8×4=32)**

15. Prove that if u is an odd vertex in a connected graph G then there must be a path in G from u to another odd vertex v of G.

16. Let G be a k-regular graph, where k is an odd number. Prove that the number of edges in G is a multiple of k.

17. Given any two vertices u and v of a graph G, every u – v walk contains a u – v path.

18. If T is a tree with n vertices then prove that it has precisely n – 1 vertices.

19. Let G be a connected graph. Then G is a tree if and only if every edge of G is a bridge.

20. Let v be a vertex of the connected graph G. Then prove that v is a cut vertex of G if and only if there are two vertices u and w of G, both different from v, such that v is on every u – w path in G.

21. Explain Konigsberg Bridge problem and its graph model.

22. Prove that a simple graph G is Hamiltonian if and only if its closure c(G) is Hamiltonian.

23. Define Hamiltonian graphs. Let G be a simple graph with n vertices and let u and v be non-adjacent vertices in G such that d(u) + d(v) ≥ n. Then prove that G is Hamiltonian if and only if G + uv Hamiltonian.

24. Prove that a graph G is planar if and only if it has no sub-graph isomorphic to a sub-division of K_5 or $K_{3,3}$

25. Let G be a graph. Then prove that $\chi(G) \geq 3$ if and only if G has an odd cycle.

26. Let G be a nonempty graph. Then prove that $\chi(G) = 2$ if and only if G is bipartite.

PART – D

Answer **any 2** questions from this Part. **Each** question carries **6** marks. **(2×6=12)**

27. Explain the Matrix representation of graphs with example.

28. Prove that a graph G is connected if and only if it has a spanning tree.

29. Let G be a graph with n vertices and q edges. Then prove that G has at least $n - \omega(G)$ edges.

30. State and prove Euler's formula for planar graphs.