

Third Semester FYUGP Degree (Reg) Examination**NOVEMBER 2025****KU3DSCSTA201 - PROBABILITY THEORY**

2024 Admission onwards

Time : 1.5 hours

Maximum Marks : 50

Section A**Answer any 6 questions. Each carry 2 marks.**

1. Define the term probability.
2. Define sample space and event with example.
3. What are the properties of p.m.f. of a random variable?
4. Define random variable.
5. Give an example of a random variable whose m.g.f. does not exist.
6. Prove that $\varphi_{cX}(t) = \varphi_X(ct)$.
7. Let X be a continuous r.v with the following pdf. Find $E(X)$.

$$f(x) = \begin{cases} \frac{1}{2} & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

8. What do you mean by "moments about a point A"?

Section B**Answer any 4 questions. Each carry 6 marks.**

9. Let the r.v. X assumes the value 'r' with the probability law: $P(X = r) = q^{r-1}p; r = 1, 2, \dots$. Find the m.g.f of X and hence its mean and variance.
10. Let X have m.g.f $M_X(t)$. Define $K_X(t) = \log M_X(t)$. Find $\frac{d^2}{dt^2}(K_X(t))$.
11. Show that the characteristic function of the sum of two independent variables is equal to the product of their characteristic functions.
12. Let X and Y be two random variables each taking three values $-1, 0, 1$ and having the joint probability distribution given below:

$Y \backslash X$	-1	0	1	$P(Y)$
-1	0	0.1	0.1	0.2
0	0.2	0.2	0.2	0.6
1	0	0.1	0.1	0.2
$P(X)$	0.2	0.4	0.4	1

Compute $E(X + Y)$, $E(X)$, and $E(Y)$.

13. Let the joint probability density function of the random variables X and Y be:

$$f(x, y) = \begin{cases} 2(x + y - 3xy^2), & 0 < x < 1, 0 < y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Find (a) $E[X | Y = y]$ and (b) $V[X | Y = y]$.

14. Define conditional expectation and state its properties.

Section C

Answer any 1 questions. Each carry 14 marks.

15. Two fair dice are rolled. Find the probability that
- (i) Both the dice show 2
 - (ii) The sum of numbers obtained is 6 or 10
 - (iii) The sum of the numbers obtained is less than 10
 - (iv) The sum is divisible by 3
 - (v) Both the dice show prime numbers.

16. A random variable X has the following p.m.f.

x:	0	1	2	3	4	5	6	7	8
f(x):	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$	$15k$	$17k$

Determine

- (i) k
- (ii) $P(X < 3)$
- (iii) $P(X \geq 3)$
- (iv) $P(0 < X < 5)$