



K25P 1901

Reg. No. :

Name :

II Semester M.Sc. Degree (C.B.C.S.S.-OBE – Reg./Supple./Imp.)

Examination, April 2025

(2023 and 2024 Admissions)

MATHEMATICS

MSMAT02C09/MSMAF02C09 : Advanced Topology

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **any five** questions. **Each** question carries **4** marks.

1. Define compact space. Prove that the real line $\mathbb{R}$ is not compact.
2. Prove that every closed interval in $\mathbb{R}$ is uncountable.
3. Prove that a subspace of a first countable space is first countable.
4. Prove that the space $\mathbb{R}_l$ is normal.
5. Prove or disprove : A completely regular space is regular.
6. Let $A \subset X$; let $f: A \rightarrow \mathbb{Z}$ be a continuous map of A into the Hausdorff space $\mathbb{Z}$. Then prove that there is at most one extension of f to a continuous function $g: \bar{A} \rightarrow \mathbb{Z}$.
(5×4=20)

PART – B

Answer **any three** questions. **Each** question carries **7** marks.

7. Prove that every nonempty finite ordered set has the order type of a section $\{1, \dots, n\}$ of $\mathbb{Z}_+$, so it is well-ordered.
8. Let X be a metrizable space. If X is limit point compact prove that X is sequentially compact.
9. Prove that the space $\mathbb{R}_l$ satisfies all the countability axioms but the second.

P.T.O.



10. Prove that the Sorgenfrey plane $\mathbb{R}_l^2$ is not normal.
11. Let X be a completely regular space. If Y_1 and Y_2 are two compactifications of X satisfying the extension property, then prove that Y_1 and Y_2 are equivalent. (3×7=21)

PART – C

Answer **any three** questions. **Each** question carries **13** marks.

12. a) Let X be a Hausdorff space. Then prove that X is locally compact if and only if given x in X and given a neighborhood U of x , there is a neighborhood V of x such that $\bar{V}$ is compact and $\bar{V} \subset U$.
- b) State and prove the Uniform Continuity Theorem.
13. a) Prove that every regular space with a countable basis is normal.
- b) Prove that every metrizable space is normal.
14. State and prove Urysohn lemma.
15. State and prove Tietze Extension Theorem.
16. Let X be a set ; let $\mathcal{D}$ be a collection of subsets of X that is maximal with respect to the finite intersection property. Then prove the following :
- a) Any finite intersection of elements of $\mathcal{D}$ is an element of $\mathcal{D}$.
- b) If A is a subset of X that intersects every element of $\mathcal{D}$, then A is an element of $\mathcal{D}$. (3×13=39)
-