K25P 0955

Reg. No. : ....

Name : ....

IV Semester M.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular) Examination, April 2025
(2023 Admission)
MATHEMATICS
MSMAT04C15 : Complex Function Theory

Time : 3 Hours | Max. Marks : 80

PART – A

Answer **any five** questions from this Part. **Each** question carries **4** marks. **(5×4=20)**

1. Define the function $E_p(z)$ for p = 0, 1, 2, . . .

2. Prove that $\lim_{z\to 0} \frac{\log(1+z)}{z} = 1$.

3. Define the Riemann zeta function and establish the relation between Riemann zeta function and gamma function.

4. Show that C and D = {z : |z| < 1} are homeomorphic.

5. State Schwaez Reflection principle.

6. With the usual notations prove that $P_r(\theta) = \text{Re}\left(\frac{1+re^{i\theta}}{1-re^{i\theta}}\right)$.

PART – B

Answer **any three** questions from this Part. **Each** question carries **7** marks. **(3×7=21)**

7. Show that $\frac{1}{2}|z| \le |\log(1+z)| \le \frac{3}{2}|z|$.

8. Discuss the convergence of the infinite product $\prod_{n=1}^{\infty} \frac{1}{n^p}$ for p > 0.

9. Prove that $\zeta(z) = 2\,(2\pi)^{z-1}\Gamma\,(1-z)\,\zeta(1-z)\,\sin\left(\frac{\pi z}{2}\right)$.

P.T.O.

10. Find the simplest meromorphic function in the plane with a pole at every integer n.

11. If u is harmonic, show that $f = u_x - iu_y$ is analytic.

PART – C

Answer **any three** questions from this Part. **Each** question carries **13** marks. **(3×13=39)**

12. Prove that $\sin \pi z = \pi z \prod_{n=1}^{\infty}\left(1 - \frac{z^2}{n^2}\right)$.

13. Prove the following :

    a) $\left\{\left(1 + \frac{z}{n}\right)^n\right\}$ converges to e^z in H(C).

    b) If $G = \{z : \mathrm{Re}(z) > 0\}$ and $f_n(z) = \int_{1/n}^{n} e^{-t} t^{z-1}\, dt$ for $n \geq 1$ and Z in G, then each f_n is analytic on G and the sequence is convergent in H(G).

14. a) Prove that if $\mathrm{Re}(z) > 0$, then $\zeta(z) = \prod_{n=1}^{\infty}\left(\frac{1}{1 - p_n^{-z}}\right)$ where p_n is a sequence of prime numbers.

    b) Prove that there are infinitely many primes.

15. State and prove Mittag-Leffler's theorem.

16. Prove that the Dirchlet problem can be solved in a unit disk.