



K25P 2019

Reg. No. :

Name :

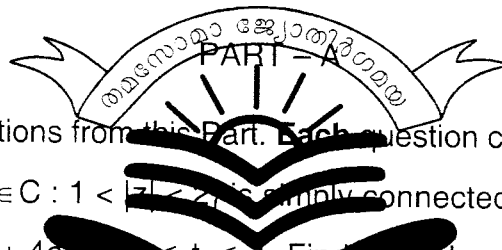
**II Semester M.Sc. Degree (CBSS – Supplementary) Examination, April 2025
(2021 and 2022 Admissions)**

MATHEMATICS

MAT 2C 09 : Foundations of Complex Analysis

Time : 3 Hours

Max. Marks : 80



Attempt **any four** questions from this Part. Each question carries **4** marks. **(4×4=16)**

1. Does the set $D = \{z \in \mathbb{C} : 1 < |z| < 2\}$ is simply connected? Justify your answer.
2. Given that $\gamma(t) = -1 + 4e^{it}$, $0 \leq t \leq 1$. Find the index of γ with respect to the point 2.
3. Prove that the function $f(z) = e^{4z}$ has an essential singularity at $z = 0$.
4. Prove that an entire function has a removable singularity at infinity if it is a constant.
5. Define the function $E_p(z/a)$. Prove that $E_p(z/a)$ has a simple zero at $z = a$.
6. Prove the following. If $\text{Re } z_n > 0$ then the product $\prod z_n$ converges absolutely if and only if the series $\sum (z_n - 1)$ converges absolutely.

Answer **any four** questions from this Part without omitting any Unit. Each question carries **16** marks.

(4×16=64)

Unit – I

7. a) Let G be a region and let f and g be analytic functions on G such that $f(z)g(z) = 0$ for all $z \in G$. Prove that either $f \equiv 0$ or $g \equiv 0$.

b) State and prove Morera's theorem.

c) Prove the following :

If $\gamma : [0, 1] \rightarrow \mathbb{C}$ is a closed rectifiable curve and $a \notin \{\gamma\}$, then $\frac{1}{2\pi i} \int_{\gamma} \frac{dz}{z-a}$ is an integer.

P.T.O.



8. Prove the following :

Let G be a connected open set and let $f : G \rightarrow \mathbb{C}$ be an analytic function. Then the following conditions are equivalent.

- a) $f \equiv 0$;
- b) there is a point a in G such that $f^n(a) = 0$ for each $n \geq 0$;
- c) $\{z \in G : f(z) = 0\}$ has a limit point in G .

9. a) State and prove Cauchy's integral formula (First Version).

- b) Find all possible values of $\int_{\gamma} \frac{dz}{1+z^2}$ where γ is any closed curve in $\mathbb{C}$ not passing through $\pm i$.

10. a) State and prove Schwarz lemma.

b) Prove the following :

If f has an isolated singularity at $z = a$ then the point $z = a$ is a removable singularity if and only if $\lim_{z \rightarrow a} (z - a)f(z) = 0$.

11. a) State and prove the Rouché's theorem.

b) State and prove the argument principle.

c) Show that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{2}}$.

12. State and prove the Laurent Series Development.

13. State and prove Arzela-Ascoli theorem.

14. a) With the usual notations, prove that $C(G, \Omega)$ is a complete metric space.

b) State and prove Montel's theorem.

15. State and prove the Riemann Mapping Theorem.
