



K25P 1902

Reg. No. :

Name :

II Semester M.Sc. Degree (C.B.C.S.S. – OBE-Reg./Supple./Imp.)

Examination, April 2025

(2023 and 2024 Admissions)

MATHEMATICS

MSMAT02C10/MSMAF02C10 : PDE and Integral Equations

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **any five** questions. **Each** question carries **4** marks.

1. Define a quasilinear equation. Give an example.
2. When an equation $L[u] = au_{xx} + 2bu_{xy} + cu_{yy} + du_x + eu_y + fu = g$ is elliptic. Give an example of an elliptic equation.
3. Let $u(x, t)$ be the solution of the Cauchy problem

$$u_{tt} - 9u_{xx} = 0, \quad -\infty < x < \infty, \quad t > 0,$$

$$u(x, 0) = f(x) = \begin{cases} 1 & \text{if } |x| \leq 2, \\ 0 & \text{if } |x| > 2. \end{cases}$$

with initial conditions

$$u_t(x, 0) = g(x) = \begin{cases} 1 & \text{if } |x| \leq 2, \\ 0 & \text{if } |x| > 2 \end{cases}$$

Find $u\left(0, \frac{1}{6}\right)$.

4. Define Neumann problem.
5. Define linear integral equation. Give an example.

6. Transform the differential equation with initial conditions $\left. \begin{aligned} \frac{d^2 y}{dx^2} - \lambda y &= f(x), \\ y(0) &= 1, y'(0) = 0 \end{aligned} \right\}$ into an integral equation.

(5×4=20)

P.T.O.



PART – B

Answer **any three** questions. **Each** question carries **7** marks.

7. Prove the following :

$u_x = c_0 u + c_1$ where c_0 is a constant and c_1 is a function of x and y has infinitely many solutions when $c_1 = 0$ and $u(x, 0) = 2e^{c_0 x}$.

8. Find the characteristic equations of the eikonal equation

$$F(x, y, u, p, q) = p^2 + q^2 - n_0^2 = 0.$$

9. Prove that the Cauchy problem $u_{tt} - c^2 u_{xx} = F(x, t)$, $-\infty < x < \infty$, $t > 0$, with initial conditions $u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$, $-\infty < x < \infty$, admits at most one solution.

10. Solve the problem $u_{tt} - 4u_{xx} = 0$, $0 < x < 1$, $t > 0$, with boundary conditions $u_x(0, t) = u_x(1, t) = 0$, $t \geq 0$, and initial conditions $u(x, 0) = f(x) = \cos^2(\pi x)$, $0 \leq x \leq 1$, $u_t(x, 0) = g(x) = \sin^2(\pi x) \cos(\pi x)$, $0 \leq x \leq 1$.

11. Prove that $\int_a^x \int_a^x \dots \int_a^x f(x) dx \dots dx = \frac{1}{(n-1)!} \int_a^x (x-\xi)^{n-1} f(\xi) d\xi$. (3×7=21)

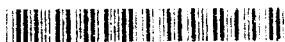
PART – C

Answer **any three** questions. **Each** question carries **13** marks.

12. a) Solve the equation $u_x + u_y = 2$ subject to the initial condition $u(x, 0) = x^2$.

b) Consider the Cauchy problem $u_x + u_y = 1$, $u(x, x) = x$. Show that it has infinitely many solutions.

13. Prove the following result : Assume that the coefficients of the quasilinear equation $a(x, y, u)u_x + b(x, y, u)u_y = c(x, y, u)$ are smooth functions of their variables in a neighborhood of the initial curve $x(0, s) = x_0(s)$, $y(0, s) = y_0(s)$, $u(0, s) = u_0(s)$.



Assume further that the transversality condition holds at each point s in the interval $(s_0 - 2\delta, s_0 + 2\delta)$ on the initial curve. Then prove that the Cauchy problem has a unique solution in the neighborhood $(t, s) \in (-\varepsilon, \varepsilon) \times (s_0 - \delta, s_0 + \delta)$ of the initial curve. If the transversality condition does not hold for an interval of s values, then prove that the Cauchy problem has either no solution at all, or it has infinitely many solutions.

14. a) Prove that the type of a linear second-order PDE in two variables is invariant under a change of coordinates. In other words, the type of the equation is an intrinsic property of the equation and is independent of the particular coordinate system used.

- b) Consider the Tricomi equation : $u_{xx} + xu_{yy} = 0$ $x > 0$. Find a canonical transformation $q = q(x, y)$, $r = r(x, y)$ and the corresponding canonical form.

15. a) Prove that a necessary condition for the existence of a solution to the Neumann problem is $\int_{\partial D} g(x(s), y(s)) ds = \int_D F(x, y) dx dy$, where $(x(s), y(s))$ is a parameterization of ∂D .

- b) Let D be a planar domain, let u be a harmonic function there, and let (x_0, y_0) be a point in D . Assume that B_R is a disk of radius R centered at (x_0, y_0) , fully contained in D . For any $r > 0$, set $C_r = \partial B_r$. Then prove that the value of u at (x_0, y_0) is the average of the values of u on the circle C_R :

$$u(x_0, y_0) = \frac{1}{2\pi R} \int_{C_R} u(x(s), y(s)) ds = \frac{1}{2\pi} \int_0^{2\pi} u(x_0 + R \cos \theta, y_0 + R \sin \theta) d\theta.$$

16. a) Prove that the relation $y(x) = \int_a^x G(x, \xi) \Phi(\xi) d\xi$ where

$$G(x, \xi) = \begin{cases} -\frac{1}{A} u(x)v(\xi) & \text{when } x < \xi, \\ -\frac{1}{A} u(\xi)v(x) & \text{when } x > \xi, \end{cases}$$

implies the differential equation, $Ly + \Phi(\xi) = 0$.

- b) Prove that $y(x) = \lambda \int_0^1 (1 - 3x\xi)y(\xi)d\xi$ has only the trivial solution if and only if $\lambda \neq \pm 2$.

(3×13=39)