



**K24P 3125**

Reg. No. : .....

Name : .....

**III Semester M.Sc. Degree (C.B.C.S.S. – OBE – Regular)**  
**Examination, October 2024**  
**(2023 Admission)**

**MATHEMATICS/MATHEMATICS (MULTIVARIATE CALCULUS AND**  
**MATHEMATICAL ANALYSIS, MODELLING AND SIMULATION, FINANCIAL**  
**RISK MANAGEMENT)**

**MSMAT03C12/MSMAF03C12 – Complex Analysis**

Time : 3 Hours

Max. Marks : 80

**PART – A**

Answer **any five** questions. **Each** question carries **4** marks.

1. Let  $\gamma$  be a rectifiable curve in  $\mathbb{C}$  and suppose that  $F_n$  and  $F$  are continuous functions on  $\{\gamma\}$ . If  $F : u \rightarrow \lim F_n$  on  $\{\gamma\}$ , prove that  $\int_{\gamma} F = \lim \int_{\gamma} F_n$ .
2. State and prove Cauchy's Estimate.
3. a) Define an entire function. Give an example.  
b) State Liouville's Theorem.
4. Suppose  $f : G \rightarrow \mathbb{C}$  is one-one, analytic and  $f(G) = \Omega$ , prove that  $f^{-1} : \Omega \rightarrow G$  is analytic and  $(f^{-1})^{-1}(\omega) = [f'(z)]^{-1}$  where  $\omega = f(z)$ .
5. a) Define meromorphic functions.  
b) State argument principle.
6. State and prove maximum modulus theorem-first version. (5×4=20)

**PART – B**

Answer **any three** questions. **Each** question carries **7** marks.

7. Let  $f$  be analytic in the disc  $B(a, R)$  and suppose that  $\gamma$  is a closed rectifiable curve in  $B(a, R)$ , prove that  $\int_{\gamma} f = 0$ .

P.T.O.



8. State and prove fundamental Theorem of Algebra.
9. State and prove Open Mapping Theorem.
10. If  $(S, d)$  is a metric space, prove that  $\mu(s, t) = \frac{d(s, t)}{1 + d(s, t)}$  is also a metric on  $S$ .
11. Prove : If  $\{f_n\}$  is a sequence in  $H(G)$  and  $f$  belongs to  $C(G, \mathbb{C})$  such that  $f_n \rightarrow f$ , then  $f$  is analytic and  $f_n^{(k)} \rightarrow f^{(k)}$  for each integer  $k \geq 1$ . (3×7=21)

### PART – C

Answer **any three** questions. **Each** question carries **13** marks.

12. a) Let  $f : G \rightarrow \mathbb{C}$  be analytic and suppose  $\bar{B}(a, r) \subset G$  ( $r > 0$ ). If  $\gamma(t) = a + re^{it}$ ,  $0 \leq t \leq 2\pi$ , then prove that  $f(z) = \frac{1}{2\pi i} \int_{\gamma} \frac{f(\omega)}{\omega - z} d\omega$  for  $|z - a| < r$ .  
 b) Find all entire functions  $f$  such that  $f(x) = e^x$  for  $x$  in  $\mathbb{R}$ .
13. Let  $G$  be an open set and let  $f : G \rightarrow \mathbb{C}$  be differentiable function, prove that  $f$  is analytic on  $G$ .
14. a) State and prove Rouché's Theorem.  
 b) Briefly explain Argument principle.
15. a) Prove that  $C(G, \mathbb{C})$  is a complete metric space.  
 b) Define a totally bounded subset of  $H(G)$ .
16. a) Define conformally equivalent region.  
 b) State and prove Riemann mapping. (3×13=39)

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