



K24P 3340

Reg. No. :

Name :

**III Semester M.Sc. Degree (CBSS – Supple./Imp.) Examination, October 2024
(2021 and 2022 Admissions)**

MATHEMATICS

MAT3C12 : Functional Analysis

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **four** questions from this Part. **Each** question carries **4** marks.

1. Give a continuous function vanishing at infinity, but does not have a compact support and prove your claim.
2. Show that c_{00} is not a closed subspace of l^∞ .
3. Show that a closed map on a metric space need not be continuous.
4. State uniform boundedness principle.
5. Prove that a linear space is uniformly convex in the norm induced by an inner product.
6. Prove that the norm in l^∞ cannot be obtained from an inner product. **(4×4=16)**

PART – B

Answer **four** questions from this Part without omitting **any** Unit. **Each** question carries **16** marks.

Unit – I

7. a) Show that for $1 \leq p < \infty$, $c_{00} \subset l^p \subset c_0 \subset c \subset l^\infty$, where all the inclusions are proper.
b) If X is a subspace of $B(T)$ with the sup norm, $1 \in X$ and f be a continuous linear functional such that $\|f\| = f(1)$ then show that f is positive.

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K24P 3340



8. a) If X is a normed space and Y is a subspace of X which is not a hyper space in X , then show that the complement Y^c is connected.
b) State and prove Hahn-Banach extension Theorem.
9. a) Let X and Y be normed spaces and $X \neq \{0\}$. Show that $BL(X, Y)$ is a Banach space in the operator norm if and only if Y is a Banach space.
b) Prove that a normed space can be embedded as a dense subspace of a Banach space.

Unit – II

10. a) State and prove Resonance Theorem.
b) Let X and Y be normed spaces and $F : X \rightarrow Y$ be linear spaces. Show that F is continuous if and only if $g \circ F$ is continuous for every $g \in Y'$.
11. a) State and prove Open Mapping Theorem.
b) State Closed Graph Theorem and show by an example that Closed Graph Theorem may not hold if the normed spaces X and Y are not Banach spaces.
12. Prove that the coefficient functionals corresponding to a Schauder basis for a Banach space X are continuous.

Unit – III

13. a) State and prove Bessel's inequality.
b) Let $\{u_\alpha\}$ be an orthonormal set in a Hilbert space H . Assume that if $x \in H$ and $\langle x, u_\alpha \rangle = 0$ for all α , then $x = 0$. Prove that $\{u_\alpha\}$ is an orthonormal basis for H .
14. a) If a non-zero Hilbert space H over K has a countable orthonormal basis, then prove that H is linearly isometric to K^n for some n , or l^2 .
b) If E is a subset of an inner product space X and $x \in \bar{E}$, then show that there exists a best approximation from E to x if and only if $x \in E$.
15. a) State and prove unique Hahn Banach extension theorem.
b) State Projection theorem.

(4×16=64)