K24P 1108

Reg. No. : ..................................

Name : ......................................

**Second Semester M.Sc. Degree (CBCSS – OBE – Regular) Examination, April 2024**
**(2023 Admission)**
**MATHEMATICS**
**MSMAT02C10/MSMAF02C10 : PDE and Integral Equations**

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any 5** questions from the following **6** questions. **Each** question carries **4** marks.

1. Show that $u(x, y) = e^{c_0x}\left[\int_0^x e^{-c_0\xi}c_1(\xi, y)d\xi + T(y)\right]$ where T(y) is determined by the initial condition is general solution of the equation $u_x = c_0u + c_1$.

2. What do you mean by the term 'System of Characteristic equations'. Explain.

3. Prove that the equation $x^2u_{xx} - 2xyu_{xy} + y^2u_{yy} = 0$ is parabolic.

4. Consider the equation $u_{xx} - 6u_{xy} + 9u_{yy} = xy^2$. Find a coordinates system (s, t) in which the equation has the form $9u_{tt} = \frac{1}{3}(s - t)t^2$.

5. Explain a Poisson's equation. Give an example.

6. What is a volterra equation of the second kind ? Give an example. **(5×4=20)**

P.T.O.

PART – B

Answer **any 3** questions from the following **5** questions. **Each** question carries **7** marks.

7. Using Lagrange Method, solve $-yu_x + xu_y = 0$.

8. Find the canonical form of the wave equation

   $u_{tt} = c^2 u_{xx}, -\infty \leq a < x < b \leq \infty, t > 0$.

9. Write the cauchy problem for the non homogeneous wave equation. Show that the cauchy problem for the non homogeneous wave equation admits at most one equation.

10. Show that a necessary condition for the existence of a solution to the Neumann problem is $\int_{\partial D} g(x(s), y(s)) = \int_D F(x, y)dxdy$ where $(x(s), y(s))$ is a parametrization of $\partial D$.

11. Transform of the equation $\frac{d^2y}{dx^2} + \lambda y = 0$, $y(0) = 0$, $y(l) = 0$ in to a Fredholm Integral Equation.

**(3×7=21)**

PART – C

Answer **any 3** questions from the following **5** questions. **Each** question carries **13** marks.

12. State and Prove The Existence and Uniqueness Theorem.

13. a) Prove the following : Suppose that $au_{xx} + 2bu_{xy} + cu_{yy} + du_x + eu_y + fu = g$ is hyperbolic in a domain D. Then there exist a coordinate system $(\xi, \eta)$ in which the equation has the canonical form $w_{\xi\eta} + l_1[w] = G(\xi, \eta)$, where $w(\xi, \eta) = u(x(\xi, \eta), y(\xi, \eta))$, $l_1$ is a first order linear differential operator, and G is a function which depend on $au_{xx} + 2bu_{xy} + cu_{yy} + du_x + eu_y + fu = g$.

    b) Consider the equation

    $u_{xx} - 2\sin x \; u_{xy} - \cos^2 x \; u_{yy} - \cos x \; u_y = 0$.

    Find a coordinate system $s = s(x,y)$, $t = t(x,y)$ that transforms the equation in to its canonical form. Show that in this coordinate system the equation has the form $u_{st} = 0$, and find the general solution.

14\. a) Consider the Dirichlet problem in a bounded domain :

$\Delta u = f(x, y), \ (x, y) \in D,$

$u(x, y) = g(x, y), \ (x, y) \in \partial D.$

Prove that the problem has at most one solution in $C^2(D) \cap C(\bar{D})$.

b) Let D be a smooth domain. Then prove the following :

i) The Dirichlet problem has at most one solution.

ii) If $\alpha \geq 0$, then the problem of the third kind has at most one solution.

iii) If u solves the Neumann problem, then any other solution is of the problem $v = u + c$, where c is a real number.

15\. a) Show that the characteristic values of $\lambda$ for the equation $y(x) = \lambda \int_0^{2\pi} \sin(x + \xi) y(\xi) d\xi$ are $\lambda_1 = 1/\pi$ and $\lambda_2 = -1/\pi$, with corresponding characteristic functions of the form $y_1(x) = \sin x + \cos x$ and $y_2(x) = \sin x - \cos x$.

b) Obtain the most general form of the equation $y(x) = \lambda \int_0^{2\pi} \sin(x + \xi) y(\xi) d\xi + F(x)$ when $F(x) = x$ and when $F(x) = 1$, under the assumption that $\lambda \neq \pm 1/\pi$.

16\. Consider the integral equation $y(x) = \lambda \int_0^1 x\xi y(\xi) d\xi + 1$.

i) Show that the iterative procedure will converge when $|\lambda| < 3$.

ii) Show that the iterative procedure leads formally to the expression

$$y(x) = 1 + x\left(\frac{\lambda}{2} + \frac{\lambda^2}{6} + \frac{\lambda^3}{18} + \ldots\right)$$

iii) Show that the exact solution of the problem is $y(x) = 1 + \dfrac{3\lambda x}{2(3 - \lambda)}$ $(\lambda \neq 3)$.

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