K24P 1106

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree (CBCSS – OBE – Regular) Examination, April 2024
(2023 Admission)
MATHEMATICS
MSMAT02C08 : Advanced Real Analysis

Time : 3 Hours | Max. Marks : 80

PART – A

Answer **five** questions from this Part. **Each** question carries **4** marks. **(5×4=20)**

1. Define pointwise convergence of a sequence of functions. For $m = 1, 2, \ldots$ $n = 1, 2, 3, \ldots$ let $s_{m,n} = \frac{m}{m+n}$. Find $\lim\limits_{n \to \infty} \lim\limits_{m \to \infty} s_{m,n}$.

2. Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.

3. Let $E(z) = \sum\limits_{n=0}^{\infty} \frac{z^n}{n!}$. Prove that $E(z + w) = E(z) + E(w)$, $z, w \in \mathbb{C}$.

4. If f is continuous with period 2π and if $\epsilon > 0$, prove that there is a trigonometric polynomial p such that $|p(x) - f(x)| < \epsilon$ for all real x.

5. Define norm of a linear transformation. If $A, B \in L(\mathbb{R}^n, \mathbb{R}^m)$, then prove that $\|A + B\| \leq \|A\| + \|B\|$.

6. If $f(0, 0) = 0$ and $f(x, y) = \frac{xy}{x^2 + y^2}$ if $(x, y) \neq (0, 0)$, find $D_1 f(0, 0)$ and $D_2 f(0, 0)$.

PART – B

Answer **three** questions from this Part. **Each** question carries **7** marks. **(3×7=21)**

7. State and prove Cauchy criterion for uniform convergence of a sequence of functions.

8. Prove that there exists a real valued function on the real line which is nowhere differentiable.

P.T.O.

9. If K is compact, $f_n \in$ (K) for n = 1, 2, 3, ... and if $\{f_n\}$ is pointwise bounded and equicontinuous on K, then prove that $\{f_n\}$ contains a uniformly convergent subsequence.

10. Suppose $a_0, a_1, \ldots, a_n$, are complex numbers, $n \geq 1$, $a_n \neq 0$, $p(z) = \sum_{0}^{n} a_k z^k$. Then prove that p(z) = 0 for complex number z.

11. Let X be a complex metric space and if ϕ is a contraction of X into X. Prove that there exists one and only one $x \in X$ such that $\phi(x) = x$.

PART – C

Answer **three** questions from this Part. **Each** question carries **13** marks. **(3×13=39)**

12. a) Suppose $f_n \to f$ uniformly on a set E in a metric space. Let x be a limit point of E, and suppose that $\lim\limits_{t \to x} f_n(t) = A_n$, n = 1, 2, 3.... . Then prove that $\{A_n\}$ converges and $\lim\limits_{t \to x} f(t) = \lim\limits_{n \to \infty} A_n$.

    b) Give an example of a series of continuous functions with a discontinuous sum.

13. Suppose $\{f_n\}$ is a sequence of functions differentiable on [a, b] and that $\{f_n(x_0)\}$ converges for some point x_0 on [a, b]. If $\{f'_n\}$ converges uniformly on [a, b] then prove that $\{f_n\}$ converges uniformly on [a, b] to a function f and $f'(x) = \lim\limits_{n \to \infty} f'_n(x)$, $a \leq x \leq b$.

14. a) Define the following terms :

    i) algebra.

    ii) uniformly closed algebra.

    iii) uniform closure of an algebra.

    b) Let $\mathscr{B}$ be the uniform closure of an algebra $\mathscr{A}$ of bounded functions, then prove that $\mathscr{B}$ is a uniformly closed algebra.

15. State and prove Parseval's theorem.

16. State and prove inverse function theorem.
