K24P 3938

Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (C.B.C.S.S. – OBE – Reg./Supple./Imp.) Examination, October 2024
(2023 Admission Onwards)
MATHEMATICS
MSMAT01C03 : Real Analysis

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any five** questions from this Part. **Each** question carries **4** marks : **(5×4=20)**

1. Define countable sets. Prove that the set of all integers is countable.
2. State and prove the distributive law of unions and intersections.
3. Define limit of a function f. Show that if f has a limit at p, then this limit is unique.
4. Give an example of a function f which has a discontinuity of the second kind at every point x on the domain. Justify.
5. Suppose f is differentiable in (a, b). If $f'(x) \geq 0$ for all $x \in (a, b)$, then prove that f is monotonically increasing.
6. If $f_1 \in R(\alpha)$ and $f_2 \in R(\alpha)$ on [a, b], then prove that $f_1 + f_2 \in R(\alpha)$ and $\int_a^b (f_1 + f_2)\, d\alpha = \int_a^b f_1\, d\alpha + \int_a^b f_2\, d\alpha$.

PART – B

Answer **any three** questions from this Part. **Each** question carries **7** marks : **(3×7=21)**

7. Let A be the set of all sequences whose elements are the digits 0 and 1. Then prove that this set A is uncountable.

P.T.O.

8. Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.
9. Give an example of a monotonic function f on (a, b) which is discontinuous at every point of a countable subset E of (a, b) and at no other point of (a, b). Justify.
10. Give an example of a function which is differentiable at all points x, but f′ is not a continuous function.
11. State and prove the fundamental theorem of calculus.

PART – C

Answer **any three** questions from this Part. **Each** question carries **13** marks :

(3×13=39)

12. If a set E in R^k has one of the following properties, then prove that it has the other two :
    a) E is closed and bounded.
    b) E is compact.
    c) Every infinite subset of E has a limit point in E.
13. Define connected sets. Prove that a subset E of the real line R^1 is connected if and only if it has the following property :
    If $x \in E$, $y \in E$ and $x < z < y$, then $z \in E$.
14. a) Let f be monotonically increasing on (a, b). Then prove that f (x +) and f(x –) exist at every point of x of (a, b) and $\sup_{a<t<x} f(t) = f(x-) \leq f(x) \leq f(x+) = \inf_{x<t<b} f(t)$. Also if $a < x < y < b$, then prove that $f(x+) \leq f(y-)$.
    b) Prove that monotonic functions have no discontinuities of the second kind.
    c) Let f be monotonic on (a, b). Then prove that the set of points of (a, b) at which f is discontinuous is at most countable.
15. State and prove L'Hospital's rule.
16. Explain rectifiable curves. If γ' is continuous on [a, b], then prove that γ is rectifiable and $\Lambda(\gamma) = \int_a^b |\gamma'(t)|\, dt$.