



K24P 3936

Reg. No. :

Name :

**I Semester M.Sc. Degree (CBCSS – OBE – Reg./Supple./Imp.)
Examination, October 2024
(2023 Admission Onwards)
Mathematics/Mathematics (Multivariate Calculus and Mathematical
Analysis, Modelling and Simulation, Financial Risk Management)
MSMAF01C01/MSMAT01C01 : ABSTRACT ALGEBRA**

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **any five** questions. **Each** question carries **4** marks.

1. List all abelian groups, upto isomorphism, of order 360.
2. List the elements of $\mathbb{Z}_2 \times \mathbb{Z}_4$. Find the order of each of the elements. Is this group cyclic ?
3. Compute all Sylow 3-subgroups of S_4 .
4. State butterfly lemma.
5. Find the center of $S_3 \times \mathbb{Z}_4$.
6. Express the polynomial $x^4 + 4$ as a product of linear factors in $\mathbb{Z}_5[x]$. **(5×4=20)**

PART – B

Answer **any three** questions. **Each** question carries **7** marks.

7. Show that every group of order p^2 is abelian.
8. State Sylow's third theorem. Prove that no group of order 30 is simple.
9. Let F be a field of quotients of D and let L be any field containing D . Prove that there exists a map $\psi : F \rightarrow L$ that gives an isomorphism of F with a subfield of L such that $\psi(a) = a$ for $a \in D$.

P.T.O.



10. State and prove second isomorphism theorem.
11. An ideal $\langle p(x) \rangle \neq \{0\}$ of $F[x]$ is maximal if and only if $p(x)$ is irreducible over F .

(3×7=21)

PART – C

Answer **any three** questions. **Each** question carries **13** marks.

12. a) Let $G_1, G_2, \dots, G_n$ be groups. Prove that $\prod_{i=1}^n G_i$ is a group under the operation $(a_1, a_2, \dots, a_n)(b_1, b_2, \dots, b_n) = (a_1b_1, a_2b_2, \dots, a_nb_n)$.
- b) Find the order of $(3, 10, 9)$ in $\mathbb{Z}_4 \times \mathbb{Z}_{12} \times \mathbb{Z}_{15}$.
- c) Let X be a G -set. For $x_1, x_2 \in X$, let $x_1 \sim x_2$ if and only if there exists $g \in G$ such that $gx_1 = x_2$. Prove that $\sim$ is an equivalence relation.
13. State and prove first Sylow theorem.
14. a) If H and K are finite subgroups of a group G , prove that $|HK| = \frac{(|H|)(|K|)}{|H \cap K|}$.
- b) Prove that any two fields of quotients of an integral domain D are isomorphic.
15. a) Prove that two subnormal series of a group G have isomorphic refinements.
- b) Prove that S_3 is solvable.
16. a) State and prove division algorithm for $F[x]$.
- b) Let $f(x) = x^4 - 3x^3 + 2x^2 + 4x - 1$ and $g(x) = x^2 - 2x + 3$ be two polynomials in $\mathbb{Z}_5[x]$. Find $q(x)$ and $r(x)$ such that $f(x)$ can be written as $f(x) = q(x)g(x) + r(x)$.

(3×13=39)