K24P 3940

Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (C.B.C.S.S. – O.B.E. – Reg./Supple./Imp.) Examination, October 2024
(2023 Admission Onwards)
MATHEMATICS/MATHEMATICS (Multivariate Calculus and Mathematical Analysis, Modelling and Simulation, Financial Risk Management)
MSMAT01C04/MSMAF01C04 : Topology

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any 5** questions from the following 6 questions. **Each** question carries **4** marks.

1. Let B be the collection of all circular regions (interiors of circles) in the plane. Show that B satisfies both conditions for a basis.
2. If $\{\tau_\alpha\}$ is a family of topologies on X. Is $\bigcup \tau_\alpha$ a topology on X ? Justify your answer.
3. Show that every order topology is Hausdorff.
4. Prove or disprove : There exist a function $f : \mathbb{R} \to \mathbb{R}$ that is continuous at precisely one point.
5. Define a quotient space and give an example.
6. Is the set of rationals $\mathbb{Q}$ connected ? Justify your answer. **(5×4=20)**

PART – B

Answer **any 3** from the following 5 questions. **Each** question carries **7** marks.

7. Let τ be a topology on a set X consisting of four sets, i.e. $\tau = \{\phi, X, A, B\}$ where A and B are non-empty proper subsets of X. What conditions must A and B satisfy ?
8. Let τ be the topology on $\mathbb{N}$ consisting of ϕ and all subsets on $\mathbb{N}$ of the form $E_n = \{n, n+1, n+2, \ldots\}$ with $n \in \mathbb{N}$.
   i) List the open sets containing the point 5
   ii) List the closed sets containing the point 5.

P.T.O.

9. Determine whether the following spaces are connected and which of them are path connected in $\mathbb{R}^2$? Give reasons.

   i) $A = \{x \times y : x^2 + y^2 = 1\} - \{1 \times 0, 0 \times 1\}$

   ii) $[0, 1] \times \{1\}$.

10. Identify the spaces that are homeomorphic to each other from the following. Give reasons.

    i) $\mathbb{Z}$ and $\mathbb{N}$

    ii) $\mathbb{R}$ and $\mathbb{R}^2$

    iii) $\{x \times y : x^2 + y^2 = 1\}$ and $\{x \times y : x^2 + y^2 \leq 1\}$.

11. Find the closure and interior of each of the following sets :

    i) I, the set of irrational in $\mathbb{R}$

    ii) $\mathbb{Q}$, the set of rationals in $\mathbb{R}$

    iii) $I \cup \mathbb{Q}$

    iv) $I \cap \mathbb{Q}$.

(3×7=21)

PART – C

Answer **any 3** from the following 5 questions. **Each** question carries **13** marks.

12. a) Prove that the topologies of $\mathbb{R}_l$ and $\mathbb{R}_K$ are strictly finer than the standard topology on $\mathbb{R}$ but are not comparable with one another.

    b) Prove that the collection

    $S = \{\pi^{-1}_1 (U) : U \text{ open in } X\} \cup \{\pi^{-1}_2 (V) : V \text{ open in } Y\}$ is a sub-basis for the product topology on $X \times Y$.

13. Prove the following :

    a) A subspace of a Hausdorff space is Hausdorff.

    b) The product of two Hausdorff spaces is Hausdorff.

    c) If $A \subset B$, then $\overline{A} \subset \overline{B}$.

14. Prove the following :

 a) Continuous image of a connected space is connected.

 b) The union of a collection of connected subspace of a topological space X that have a point in common is connected.

 c) A finite Cartesian product of connected spaces is connected.

15. Prove the following :

 a) The punctured euclidean space $\mathbb{R}^n - \{0\}$, where 0 is the origin in $\mathbb{R}^n$ is path connected for $n > 1$.

 b) The ordered square I_o^2 connected but not path connected.

16. a) Give an example that the product of two quotient maps need not be a quotient map.

 b) Prove the following : Let $g : X \to Z$ be a surjective continuous map. Let X^* be the following collection of subsets of X : $X^* = \{g^{-1}(\{z\}) : z \in Z\}$. Give X^* the quotient topology.

  i) The map g induces a bijective continuous map $f : X^* \to Z$, which is a homeomorphism if and only if g is a quotient map.

  ii) If Z is Hausdorff, so is X^*. **(3×13=39)**

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