K25U 3082

Reg. No. : ....

Name : ....

III Semester B.Sc. Honours in Mathematics Degree (CBCSS – OBE – Supplementary/Improvement) Examination, November 2025
(2021 to 2023 Admissions)
3B10 BMH : CALCULUS – III

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any four** questions out of **five** questions. **Each** question carries **one** mark.

1. Evaluate $\int_0^1 \int_1^2 dydx$.
2. State Divergence Theorem.
3. Define average value of a function of two variables on a Rectangle R.
4. Plot the point with cylindrical co-ordinate $(2, 2\pi/3, 1)$.
5. Give an example of a one-sided surface. **(4×1=4)**

SECTION – B

Answer **any 6** questions out of **9** questions. **Each** question carries **2** marks.

6. Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \sin x \cos y \, dA$.
7. Find the area enclosed by one loop for closed leaved rose $r = \cos 2\theta$.
8. Find the polar moment of inertia of a homogeneous disk D with density ρ, center at origin and radius a.
9. Describe the Surface whose equation in cylindrical co-ordinate is $z = r$.

P.T.O.

10. The point $\left(2, \frac{\pi}{3}, \frac{\pi}{4}\right)$ is given in spherical co-ordinates. Find the rectangular co-ordinates of the point.

11. Find the Gradient of $F(x, y, z) = x^2yzi + xyj + z^2k$.

12. Evaluate, $\int_C y \sin z\, ds$ .where C is the circular Helix given by the equation

$x = \cos t$, $y = \sin t$, $z = t$, $0 \leq t \leq 22\pi$.

13. If f is a function of two variable that has continuous second order partial derivatives then show that curl $(\nabla f) = 0$.

14. Identify the Surface with the Vector equation $r(u, v) = 2\cos u\, i + v\, j + 2\sin u\, k$.

(6×2=12)

SECTION – C

Answer **any 8** questions out of **12** questions. **Each** question carries **4** marks.

15. Use the mid point rule with m = n = 2, to estimate the value of the integral $\int_{x=0}^{2}\int_{y=1}^{2}(x - 3y^2)dy\, dx$.

16. Find the Volume of the solid that is bounded by the elliptic paraboloid $x^2 + 2y^2 + z = 16$, the planes $x = 2$ and $y = 2$ and the three coordinate planes.

17. Evaluate $\iint_D xy\, dA$ where D is the region bounded by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.

18. Evaluate $\iiint_E z\, dV$, where E is the solid tetrahedron bounded by the four planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$.

19. Use the change of variables $x = u^2 - v^2$, $y = 2uv$, to evaluate the integral $\iint_R y\, dA$, where R is the region bounded by the x-axis and the parabolas $y^2 = 4 - 4x$, $y^2 = 4 + 4x$, $y \geq 0$.

20. Evaluate $\int_C 2x\, ds$, where c consist of the arc c_1 of the parabola $y = x^2$, from (0, 0) to (1,1), followed by the vertical line segment c_2 from (1, 1) to (1, 2).

21. Evaluate $\oint_C y^2 dx + 3xy\, dy$, where C is the boundary of a semi annular region D in the upper half plane between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

22. Find the tangent plane to the Surface with parametric equation $x = u^2$, $y = v^2$, $z = u + 2v$ at (1, 1, 3).

23. Compute the surface integral $\iint_S x^2\, dS$, where S is the unit Sphere.

24. Evaluate $\int_C F.dr$, where $F = -y^2\, i + x\, j + z^2\, k$ and C is the curve of intersection of the plane $y + z = 2$, and the cylinder $x^2 + y^2 = 1$.

25. Find the outward flux of the Vector Field F (x, y, z) = zk, across the sphere $x^2 + y^2 + z^2 = a^2$.

26. Find the area of the part of the paraboloid $z = x^2 + y^2$, that lies under the plane $z = 9$. **(8×4=32)**

SECTION – D

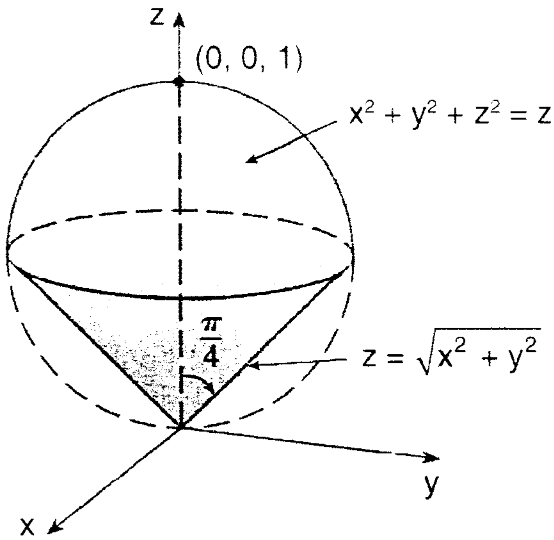
Answer **any 2** questions out of **4** questions. **Each** question carries **6** marks.

27. The joint density function of X and Y is given as

$$f(x, y) = \begin{cases} C(x + 2y), & \text{if } 0 \le x \le 10, 0 \le y \le 10 \\ 0 & \text{otherwise} \end{cases}$$. Find the value of constant C.

Then find $P(X \le 7, Y \ge 2)$.

28. Find the volume of the solid that lies above the cone $z = \sqrt{x^2 - y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$.



29. A) Show that $F(x, y, z) = y^2z^3\,i + 2xyz^3 j + 3xy^2z^2 k$ is a conservative Vector Field.

    B) Find a function f such that $\mathbf{F} = \nabla f$.

30. Using Stokes Theorem evaluate $\oint_C F.dr$, where $F = 2zi + 3xj + 5yk$, where C is the positively oriented circle $x^2 + y^2 = 4$, that forms the boundary of the surface $z = 4 - x^2 - y^2$, $z \geq 0$, in the xy plane. **(2×6=12)**