K25U 2353

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, November 2025
(2019 to 2023 Admissions)
CORE COURSE IN MATHEMATICS
5B09MAT : Vector Calculus

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any 4** questions from this Part. **Each** question carries **1** mark. **(4×1=4)**

1. Define critical point of a function $f(x, y)$.
2. Find parametric equations for the line through $(-2, 0, 4)$ parallel to $v = 2i + 4j - 2k$.
3. Find T for the circular motion $r(t) = (\cos 2t)i + (\sin 2t)j$.
4. Define the gradient field of a differentiable function $f(x, y, z)$.
5. Define an exact differential form.

PART – B

Answer **any 8** questions from this Part. **Each** question carries **2** marks. **(8×2=16)**

6. Find the gradient of $f(x, y)$ at $(-2, 1)$ where $f(x, y) = \frac{x^2}{4} + y^2$.
7. Find the linearization of $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$ at the point $(3, 2)$.
8. If $w = x^2 + y^2 + z^2$ and $z = x^2 + y^2$, find $\left(\frac{\partial w}{\partial y}\right)_z$.

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9. State the second derivative test for local extreme values of a function $f(x, y)$.
10. Find the local extreme values of $f(x, y) = x^2 + y^2 - 4y + 9$.
11. Define circle of curvature for plane curves.
12. Find the length of the curve $r(t) = (4\cos t)i + (4\sin t)j + (3t)k$, where $0 \le t \le \pi/2$.
13. If $r(t) = (t + 1)i + (t^2 - 1)j$ represents position of a particle in the xy-plane at time t, find the particles velocity and acceleration vectors at $t = 1$.
14. Find the gradient field of the function $g(x, y, z) = e^z - \ln(x^2 + y^2)$.
15. Find a parametrization of the cone $z = \sqrt{x^2 + y^2}, 0 \le z \le 1$.
16. Find the divergence of the vector field $F(x, y, z) = xi + yj + zk$.

PART – C

Answer **any 4** questions from this Part. **Each** question carries **4** marks. **(4×4=16)**

17. Find all the local maxima, local minima and saddle points of the function $f(x, y) = x^2 + xy + y^2 + 3x - 3y + 4$.
18. Find $\left(\frac{\partial w}{\partial x}\right)_{y,z}$, if $w = x^2 + y - z + \sin t$ and $x + y = t$.
19. If r is a differentiable vector function of t of constant length, then prove that $r \cdot \frac{dr}{dt} = 0$.
20. Find the length of the curve $r(t) = (\sqrt{2}t)i + (\sqrt{2}t)j + (1 - t^2)k$ from $(0, 0, 1)$ to $(\sqrt{2}, \sqrt{2}, 0)$
21. Integrate $f(x, y, z) = x - 3y^2 + z$ over the line segment C, joining the origin to the point $(1, 1, 1)$.
22. Find the flux of $F = (x - y)i + xj$ across the circle $x^2 + y^2 = 1$ in the xy plane.
23. Find the flux of $F = xyi + yzj + xzk$ outward through the surface of the cube cut from the first octant by the planes $x = 1$, $y = 1$ and $z = 1$.

PART – D

Answer **any 2** questions from this Part. **Each** question carries **6** marks. **(2×6=12)**

24. a) Find the derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $P_0(1, 1, 0)$ in the direction of $v = 2i - 3j + 6k$.

    b) In what directions does f change most rapidly at P_0 and what are the rates of change in these directions ?
25. Find the curvature for the helix $r(t) = (a\cos t)i + (a\sin t)j + (bt)k$, $a, b \ge 0$, $a^2 + b^2 \ne 0$.
26. Show that $F = (e^x\cos y + yz)i + (xz - e^x\sin y)j + (xy + z)k$ is conservative over its natural domain and find a potential function for it.
27. Find the surface area of a sphere of radius a.

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