K25U 2853

Reg. No. : ....

Name : ....

V Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/Improvement) Examination, November 2025
(2021 to 2023 Admissions)
5B22BMH : COMPLEX ANALYSIS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any four** questions from the following. **Each** question carries **1** mark. **(4×1=4)**

1. Find $\overline{z_1 z_2}$ if $z_1 = 1+2i$ and $z_2 = 2+i$.
2. Does the set $G = \{ z : |z| < 1\}$ is an open set ? Justify your answer.
3. Evaluate $\int_{-i}^{i} (z^2+1)dz$.
4. Is the sequence $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+...$ a convergent sequence ? Justify your answer.
5. Evaluate i^{101}.

SECTION – B

Answer **any six** questions. **Each** question carries **2** marks. **(6×2=12)**

6. Find the polar form of the complex number $-1+i$.
7. Show that $\sin z = \sin x \cosh y + i \cos x \cosh y$.
8. Evaluate $\arg\left(\frac{z_1}{z_2}\right)$ if $z_1 = 2+2i$ and $z_2 = 3i$.
9. Solve $\sin 2z = 0$.
10. Evaluate $\oint_C x^2 dz$ where C is the line segment joining to $-1-i$ to $1+i$.
11. Find the real part u and imaginary part v of $w = f(z) = z^2 + z - 1$.
12. Give an example of an absolutely convergent sequence.
13. Find the Laurent series of $z^2 \sin(1/z)$ with centre 0.
14. If $\lim_{z \to z_0} f(z)$ exists, then show that this limit is unique.

P.T.O.

SECTION – C

Answer **any eight** questions. **Each** question carries **4** marks. **(8×4=32)**

15. Is $f(z) = e^z$ is analytic ? Justify your answer.
16. Show that the set of values of $\ln((-1)^3)$ is differ from the set of values of $3\ln(-1)$.
17. Verify that $u = x^2 - y^2 - y$ is harmonic in the whole complex plane.
18. Show that
    (a) $|\cos z|^2 = \cos^2 x + \sinh^2 y$
    (b) $|\sin z|^2 = \sin^2 x + \sinh^2 y$.
19. Integrate $z^2 / z^2 - 1$ counter clock-wise around the circle $|z+1| = \frac{1}{100}$.
20. Show that if a sequence converges then its limit is unique.
21. State and prove the Ratio Test.
22. Test the convergence of the series $\sum_{n=0}^{\infty} \frac{(z+i)^n}{n!}$.
23. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \left[1+(-1)^n + \frac{1}{3^n}\right] z^n$.
24. Integrate $f(z) = \frac{1}{z^2+z^4}$ clockwise around the circle $|z| = \frac{\pi}{2}$.
25. Discuss the singularity of
    (a) $\sin\left(\frac{1}{z-1}\right)$
    (b) $e^{\frac{1}{2z}}$
26. Show that if a series converges absolutely, it is convergent. Is the converse true ? Justify your answer.

SECTION – D

Answer **any two** questions. **Each** question carries **6** marks. **(2×6=12)**

27. i) Find and sketch the set of points given by $|z^2+1| = |z^2-1|$.
    ii) Find the derivative of $f(z) = (z^{100}+i)^{-5}$ at $z = 0$.

28. State Cauchy's integral formula and using Cauchy's integral formula, evaluate counter clockwisely $\oint_C \left(\frac{1}{z-1} + \frac{1}{(z+2)(z-2)}\right)$ where $C : |z+1| = 4$.
29. i) Find the Maclaurian's series for $f(z) = \frac{1}{(z-1)^m}$.
    ii) Find the Taylor series for $f(z) = \frac{1}{z^2}$ about $z = -1$.
30. Evaluate the following integral, where C is the ellipse $4x^2 + y^2 = 4$, clockwise, $\oint_C \left[\frac{z^2 \sin(\pi z)}{z^2-1} + \frac{e^{1/z}}{z}\right] dz$.