Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/ Improvement) Examination, November 2025
(2019 to 2023 Admissions)
CORE COURSE IN MATHEMATICS
5B07MAT : Abstract Algebra

Time : 3 Hours | Max. Marks : 48

PART – A

Answer **any 4** questions from this Part. **Each** question carries **1** mark. **(4×1=4)**

1. Define a group.
2. State the division algorithm for Z.
3. Find all the generators of the group of integers, Z under addition.
4. Define the factor group.
5. Define homomorphism.

PART – B

Answer **any 8** questions from this Part. **Each** question carries **2** marks. **(8×2=16)**

6. Prove that in a group the left cancellation law holds.
7. Define Klein 4-group.
8. Prove that every cyclic group is abelian.
9. Find the cyclic subgroup generated by 3 in Z_4.
10. Prove that the collection of all positive nonzero rational numbers with the operation $*$ defined as $a * b = (ab)/2$ is an abelian group.

11. Suppose H and K are subgroups of a group G such that $K \leq H \leq G$ and suppose (H : K) and (G : H) are both finite. Then prove that (G : K) is finite and (G : K) = (G : H) (H : K).
12. Find the left cosets of the subgroup 3Z of the group Z under addition.
13. State and prove Lagranges theorem.
14. Express the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 2 & 6 & 3 & 7 & 4 & 5 & 1 \end{pmatrix}$ of {1, 2, 3, 4, 5, 6, 7, 8} as a product of disjoint cycles.
15. If R is a ring with unity 1, then prove that R has characteristic $n > 0$ if and only if n is the smallest positive integer such that $n. 1 = 0$.
16. If R is a ring with additive identity 0, then for any $a, b \in R$ prove that $(-a)(-b) = ab$.

PART – C

Answer **any 4** questions from this Part. **Each** question carries **4** marks. **(4×4=16)**

17. If a is a generator of a finite cyclic group G of order n, then prove that the other generators of G are the elements of the form a^r, where r is relatively prime to n.
18. Draw the lattice diagram of Z_{18}.
19. Suppose G and G′ are groups and $\phi : G \to G'$ is a one to one function such that $\phi(xy) = \phi(x)\,\phi(y), \forall x, y \in G$, then prove that $\phi(G)$ is a subgroup of G′.
20. Prove that order of an element of a finite group divides the order of the group.
21. Compute the product (4 7 8) (2 1) (7 2 8 1 5), which are permutations of {1, 2, 3, 4, 5, 6, 7, 8}.
22. Prove that a homomorphism ϕ from a group G into a group G′ is a one to one function if and only if the kernal of ϕ is {e} ,where e is the identity element in G.
23. If $\phi : G \to G'$ is a homomorphism, then prove that for $a \in G$, $\phi(a^{-1}) = (\phi(a))^{-1}$.

PART – D

Answer **any 2** questions from this Part. **Each** question carries **6** marks. **(2×6=12)**

24. a) Prove that subgroup of a cyclic group is cyclic.
    b) Prove that subgroups of Z under addition are precisely the groups nZ under addition for $n \in Z$.
25. Prove that every group is isomorphic to a group of permutations.
26. Let H be a subgroup of a group G. Then the coset multiplication is well defined by the equation (aH) (bH) = (ab) H if and only if H is a normal subgroup of G.
27. a) Prove that every field is an integral domain.
    b) Prove that every finite integral domain is a field.

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