K25U 2852

Reg. No. : ....

Name : ....

V Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/Improvement) Examination, November 2025
(2021 to 2023 Admissions)
5B21 BMH : ADVANCED LINEAR ALGEBRA

Time : 3 Hours   Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.

1. Define characteristic equation.
2. Define geometric multiplicity of an eigenvalue.
3. Prove that $P = \begin{bmatrix} 3/5 & 4/5 \\ -4/5 & 3/5 \end{bmatrix}$ is orthogonal.
4. State Fundamental theorem of algebra.
5. Define orthogonal vectors.   **(4×1=4)**

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. Find the eigenvalues of $A = \begin{bmatrix} 7 & -15 \\ 2 & -4 \end{bmatrix}$.
7. Prove that the determinant of an $n \times n$ matrix A is equal to the product of eigenvalues.
8. Prove that similar matrices have the same characteristic polynomial.
9. Prove that in any inner product space V, $||x + y||^2 + ||x - y||^2 = 2||x||^2 + 2||y||^2$.

10. Let P be an orthogonal $n \times n$ matrix and T be the linear transformation defined by $T(x) = Px$. Using standard inner product, show that for any $x, y \in \mathbb{R}^n$, $\langle T(x), T(y) \rangle = \langle x, y \rangle$.
11. Let U and W be subspaces of a vector space V. Prove that $U + W = \{u + w / u \in U, w \in W\}$ is a subspace of V.
12. Suppose S is a subspace of $\mathbb{R}^n$. Prove that $S = (S^{\perp})^{\perp}$.
13. Prove that eigenvalues of a Hermitian matrix are real.
14. If a matrix A is unitarily diagonalisable, then prove that A is normal.   **(6×2=12)**

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. Find eigenvectors for the matrix $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$.
16. Prove that eigenvectors corresponding to different eigenvalues are linearly independent.
17. Suppose A is a diagonalisable matrix. Then prove that all its eigenvalues are real numbers and for each eigenvalue, the geometric multiplicity equals the algebraic multiplicity.
18. Use the Gram Schmidt process to find an orthonormal basis for the subspace of $\mathbb{R}^4$ spanned by the vectors $v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}$ and $v_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix}$.
19. Let A be a symmetric matrix. Prove that the eigenvectors corresponding to distinct eigenvalues of A are orthogonal.
20. Find the symmetric matrix A such that the quadratic form $f(x, y, z) = 3x^2 + 4xy + 2y^2 + 5z^2 - 2xz + 2yz$ can be expressed as $X^T AX$. Determine the signs of the eigenvalues of A.
21. Suppose that A is an $m \times n$ real matrix. Prove that $R(A)^{\perp} = N(A^T)$.

22. Let V be a finite dimensional inner product space and S be any subspace of V. Prove that $V = S \oplus S^{\perp}$.
23. Determine the orthogonal projection of $\mathbb{R}^3$ onto $U = \mathrm{Lin}\{(1, 2, -1)^T, (1, 0, 1)^T\}$.
24. Let A be a Hermitian matrix. Prove that the eigenvalues of A are real.
25. State and prove a characterisation for a unitary matrix.
26. State and prove spectral decomposition theorem.   **(8×4=32)**

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. Diagonalise the matrix $A = \begin{bmatrix} 3 & -1 & 2 \\ 5 & -3 & 5 \\ 1 & -1 & 2 \end{bmatrix}$.
28. Sketch the curve $3x^2 + 4xy + 6y^2 = 14$ in the xy plane.
29. Suppose A is an $m \times n$ real matrix of rank n. Prove that $P = A(A^TA)^{-1}A^T$ is an orthogonal projection of R^m onto the range R(A) of A.
30. Let $A = \begin{bmatrix} 7 & 0 & 9 \\ 0 & 2 & 0 \\ 9 & 0 & 7 \end{bmatrix}$. Express A in the form $A = \lambda_1 E_1 + \lambda_2 E_2 + \lambda_3 E_3$, where $\lambda_1, \lambda_2, \lambda_3$ are eigenvalues of A and E_1, E_2, E_3 are symmetric idempotent matrices such that if $i \neq j$, then E_iE_j is the zero matrix. Determine a matrix B such that $B^3 = A$.   **(2×6=12)**

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