K25U 2851

Reg. No. : ....

Name : ....

V Semester B.Sc. Honours in Mathematics Degree (CBCSS – OBE – Regular/Supplementary/Improvement) Examination, November 2025 (2021 to 2023 Admissions)

5B20BMH : INTEGRAL TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any four** questions from the following. **Each** question carries **1** mark.

1. Find the Laplace transform of the function $f(t) = 1/2, t \geq 0$.
2. Define the unit impulse function.
3. Define the fundamental period of a function f(x).
4. Show that $\int_{-\pi}^{\pi} \sin mx \cos nx = 0$ if $m \neq n$.
5. Give an example of a two dimensional partial differential equation. **(4×1=4)**

SECTION – B

Answer **any six** questions. **Each** question carries **2** marks.

6. Given that $f(t) = t^2 - 2t + 7 \cos wt$. Find F(s).
7. Find the Laplace transform of sin h at.
8. Find the Fourier coefficient a_0 for the function $f(x) = x^2, -\pi < x < \pi$ with $f(x + 2\pi) = f(x)$.
9. Does the constant function is a periodic function ? Justify your answer.

P.T.O.

10. Does the function $f(x) = x^2 \cos x$ is even ? Justify your answer.
11. f(x) has period p, show that f(ax), $a \neq 0$ has period p/a.
12. What is a Fourier Cosine Integral ? Explain.
13. Solve $u_{xx} = 3u_x$ like an ODE in x.
14. Show that the function $u(x, t) = 5x^2 + 3t^2$ is a solution of the one-dimensional wave equation $u_{tt} = c^2 u_{xx}$ for a suitable value of c. Find the value of c. **(6×2=12)**

SECTION – C

Answer **any eight** questions. **Each** question carries **4** marks.

15. Solve $y'' + y' + 9y = 0$, $y(0) = 0.16$, $y'(0) = 0$.
16. Find the inverse transform of $\frac{1}{s^2(s^2 + w^2)}$.
17. Show that $\mathcal{L}(\sin wt) = \frac{w}{w^2 + t^2}$.
18. Find the Fourier series of the function

$$f(x) = \begin{cases} k, & -2 < x < -0.5 \\ 0, & -0.5 < x < 0.5 \\ -k, & 0.5 < x < 2 \end{cases} \text{ with } p = 4$$

19. Find the Fourier cosine and sine transformation of the function

$$f(x) = \begin{cases} k, \text{ if } & 0 < x < a \\ 0, \text{ if } & x > a \end{cases}$$

20. Show that the complex Fourier coefficients of an even function are real.
21. Find the Fourier cosine and sine transforms of the function

$$f(x) = \begin{cases} k, 0 < x < a \\ 0, x > a \end{cases}.$$

22. With the usual notations, show that $\mathscr{F}_c\{f'(x)\} = -\sqrt{\frac{2}{\pi}} f(0) + w\mathscr{F}_s\{f(x)\}$.
23. Find the Fourier transform of $f(x) = e^{-a|x|}$ if $x < 0$ and $f(x) = 0$ if $x > 0$; where $a > 0$.
24. Verify that $u = \sin x \sin hy$ is a solution of $u_{xx} + u_{yy} = 0$. Does this equation has another set of solutions ? Explain.
25. Find the normal form and solution of the PDE $4u_{xx} + u_{yy} = 0$.
26. Find the temperature u (x, t) in a laterally insulated aluminum bar 60 cm long if the initial temperature is 120 sin (π/180)°C and the ends are kept at 0°C. How long will it take for the maximum temperature in the bar to drop to 40°C ? [Physical data for aluminum : density 2.70 gm/cm^3, specific heat 0.215 cal/(gm°C), thermal conductivity 0.49 cal/(cmsec°C)]. **(8×4=32)**

SECTION – D

Answer **any two** questions. **Each** question carries **6** marks.

27. i) Find the inverse transform of $\ln\left(\frac{s^2 + a^2}{s^2 + b^2}\right)$, $a, b > 0$.

    ii) With the usual notations, show that $\mathcal{L}(u(t - a)) = \frac{e^{-as}}{s}$.
28. Find the Fourier series of $f(x) = x \sin x$ if $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$.
29. Find the Fourier integral representation of the function $f(x) = \begin{cases} 2, |x| < 2 \\ 0, |x| > 2 \end{cases}$.
30. Derive tha d'Alembert's solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$. **(2×6=12)**

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