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K25U 2349

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/ Improvement) Examination, November 2025
(2019 to 2023 Admissions)
CORE COURSE IN MATHEMATICS
5B05 MAT : Set Theory, Theory of Equations and Complex Numbers

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any 4** out of 5 questions. **Each** question carries **1** mark : **(4×1=4)**

1. Define an infinite set.
2. If α, β, γ are the roots of $2x^3 + x^2 - 2x - 1 = 0$ then find the value of $\alpha + \beta + \gamma$.
3. Write the discriminant of the cubic equation $ax^3 + 3bx^2 + 3cx + d = 0$.
4. State triangle inequality of complex numbers.
5. Write De Moivre's formula.

PART – B

Answer **any 8** questions out of 11 questions. **Each** question carries **2** marks : **(8×2=16)**

6. State Uniqueness theorem.
7. Prove that the set $\mathbb{Z}$ of all integers is denumerable.
8. Give an example of a countable collection of finite sets whose union is not finite.
9. If q, r, s are positive, show that the equation $f(x) = x^4 + qx^2 + rx - s = 0$ has one positive, one negative and two imaginary roots.
10. If $\alpha, \beta, \gamma, \ldots$ are the roots of $x^n + p_1x^{n-1} + p_2x^{n-2} + \ldots + p_n = 0$ then find the value of $\Sigma\alpha^2$.

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11. Show that $x^5 - 2x^2 + 7 = 0$ has atleast two imaginary roots.
12. If α, β, γ are the roots of $x^3 - x - 1 = 0$, find the equation whose roots are $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$.
13. Define reciprocal equation.
14. Prove that $(x^3 + y^3 + z^3 - 3xyz)^2 = X^3 + Y^3 + Z^3 - 3XYZ$, where $X = x^2 + 2yz$, $Y = y^2 + 2xz$, $Z = z^2 + 2xy$.
15. Determine the principal value of the argument of $z = 3 + 3\sqrt{3i}$.
16. Write the polar form of a complex number $z = 1 + i$.

PART – C

Answer **any 4** questions out of 7 questions. **Each** question carries **4** marks : **(4×4=16)**

17. If A_m is a countable set for each $m \in N$ then show that the union $A = \bigcup_{m=1}^{\infty} A_m$ is countable.
18. Show that the following statements are equivalent.
    i) S is a countable set.
    ii) There exists a surjection of N onto S.
    iii) There exists an injection of S into N.
19. Find an upper limit to the roots of $f(x) = 3x^4 - 61x^3 + 127x^2 + 220x - 520$.
20. Find the condition that the sum of two roots α, β of $x^4 + p_1x^3 + p_2x^2 + p_3x + p_4 = 0$ may be zero.
21. If $\alpha, \beta, \gamma, \delta$ are the roots of $ax^4 + 4bx^3 + 6cx^2 + 4dx + c = 0$ then find the value of $\sum(\alpha - \beta)^2\gamma^2\delta^2$.
22. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$ and $S_1 = C_0 + C_3 + C_6 + \ldots$ these series being continued as far as possible. Show that the value of $S_1 = \frac{1}{3}\left(2^n + 2\cos\frac{n\pi}{3}\right)$.
23. Solve $z^n = 1$.

PART – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks : **(2×6=12)**

24. Show that the set $\mathbb{Q}$ of all rational numbers is denumerable.
25. Find an upper and lower limits to the positive roots of $f(x) = x^3 - 10x^2 - 11x - 100$.
26. Describe Cardan's method.
27. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$ and $S_3 = C_2 + C_5 + C_8 + \ldots$ these series being continued as far as possible. Show that the value of

$$S_3 = \frac{1}{3}\left(2^n + 2\cos\left(\frac{(n+2)\pi}{3}\right)\right).$$