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K25U 2857

Reg. No. : ....

Name : ....

V Semester B.Sc. Honours in Mathematics Degree(C.B.C.S.S-Supplementary) Examination, November 2025

(2019 to 2020 Admissions)

Core Course

BHM 501: SPECIAL FUNCTIONS

Time : 3 Hours | Max. Marks : 60

SECTION A

Answer any 4 questions out of 5 questions. Each question carries 1 mark.

1. Determine the nature of the point $x = 0$ for the equation $y'' + (\sin x)y = 0$.

2.Define ordinary point x_0 of general homogeneous second order linear equation $y'' + P(x)y' + Q(x)y = 0$.

3.Find $\Gamma(1)$.

4.Write Rodrigues formula.

5. Define Legendre series. (4x 1 = 4)

SECTION B

Answer any 6 questions out of 9 questions. Each question carries 2 marks.

6. State the orthogonality property of Legendre polynomials.

7.Evaluate $(\frac{5}{2})!$.

8.Find $J_{-3/2}(x)$.

9.Locate and classify singular points of differential equation $x^2y'' + (2 - x)y' = 0$.

10.Define improper integrals and classify them.

11.Determine the nature of the point $x=0$ of the equation $xy''+(\sin x)y=0$.

12.Write a short note on Legendre series.

13. Evaluate each of the following

i) $\frac{\Gamma(6)}{2\Gamma(3)}$ ii) $\frac{\Gamma(\frac{5}{2})}{\Gamma(\frac{1}{2})}$

14.State Cauchy principal value. (6x2=12)

SECTION C

Answer any 8 questions out of 12 questions. Each question carries 4 marks.

15.Find a power series solution of the form $\sum_{n=0}^{\infty} a_n x^n$ to solve the differential equation $y' = y$.

16. Show that $\Gamma(p+1) = p\,\Gamma(p)$.

17.Show that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.

18. Prove that $\frac{d}{dx}[J_0(x)] = -J_1(x)$.

19. Prove that $F(1,b,b,x) = 1 + x + x^2 + x^3 + \ldots\ldots\ldots$

20.For the differential equation $x^2(x^2 - 1)^2 y'' - x(1-x)y' + 2y = 0$, locate and classify its singular points on the x-axis.

21. Consider the generating function $\frac{1}{\sqrt{1-2xt+t^2}} = \sum_{n=0}^{\infty} p_n(x)t^n$ .By differentiating both sides with respect to t, show that

$(x-t)\sum_{n=0}^{\infty} p_n(x)t^n = (1-2xt+t^2)\sum_{n=1}^{\infty} n\, p_n(x)t^{n-1}$.

22.Find the indicial equation and its roots for the differential equation $4x^2y'' + (2x^4 - 5x)y' + (3x^2 + 2)y = 0$.

23.Show that $F'(a,b,c,x) = \frac{ab}{c}F(a+1, b+1, c+1, x)$.

24.Test the convergence of following integrals:

i) $\int_0^{\infty} \frac{x^2}{4x^4+25}\,dx$

ii) $\int_0^{\infty} \frac{x}{\sqrt{x^4+x^2+1}}\,dx$

25. Express Bessel functions $J_2(x), J_3(x)$ and $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$.

26.Prove that $B(u,v) = \frac{\Gamma(u)\Gamma(v)}{\Gamma(u+v)}$ $u, v > 0$ (8x4=32)

SECTION D

Answer any 2 questions out of 4 questions. Each question carries 6 marks.

27.State and prove orthogonal property of Bessel functions.

28.Define gamma function and prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

29.Determine the nature of the point $x = \infty$ for the Bessel's equation $x^2y'' + xy' + (x^2 - p^2)y = 0$ and find the exponents from the indicial equation.

30. Find the general solution of differential equation $x(1-x)y'' + \left(\frac{3}{2} - 2x\right)y' + 2y = 0$ at the point $x=0$. (2x6=12)