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K25U 1408

Reg. No. : ....

Name : ....

Second Semester B.Sc. (Hon's) Mathematics Degree (C.B.C.S.S. – O.B.E. – Supplementary/Improvement) Examination, April 2025
(2021 to 2023 Admissions)
Core Course
2B07 BMH : THEORY OF NUMBERS AND EQUATIONS

Time : 3 Hours Max. Marks : 60

SECTION– A

Answer **any 4** questions from this Section. **Each** question carries **1** mark. **(4×1=4)**

1. Define greatest common divisor of two integers a and b.
2. State Fermat's theorem.
3. Give an example for a pseudoprime.
4. Give an example for a reciprocal equation.
5. State Descartes' rule of signs.

SECTION– B

Answer **any 6** questions from this Section. **Each** question carries **2** marks. **(6×2=12)**

6. Using Euclidean algorithm, find the gcd of 12378 and 3054.
7. For any integer $k \neq 0$, prove that gcd (ka, kb) = |k| gcd (a,b).
8. If p is a prime and p|ab, then prove that p|a or p|b.
9. If $a \equiv b \pmod n$, then prove that $b \equiv a \pmod n$.
10. Prove that 41 divides $2^{20} - 1$.
11. Show that τ is a multiplicative function.
12. Frame an equation with rational coefficients, one of whose roots is $\sqrt{5} + \sqrt{2}$.
13. If α, β, γ are the roots of the equation $x^3 + ax^2 + bx + c = 0$, form the equation whose roots are $\alpha\beta, \beta\gamma, \gamma\alpha$.
14. Discuss the nature of the roots of the equation $x^5 - 6x^2 - 4x + 5 = 0$.

P.T.O.

SECTION– C

Answer **any 8** questions from this Section. **Each** question carries **4** marks. **(8×4=32)**

15. If n is an odd integer, prove that $n^4 + 4n^2 + 11$ is of the form 16k. p-19 ex.
16. Let a and b be integers, not both zero. Prove that a and b are relatively prime if and only if there exist integers x and y such that 1 = ax + by.
17. Solve the linear Diophantine equation 172x + 20y = 1000.
18. If a|c and b|c with gcd (a,b) = 1, then prove that ab|c.
19. If $ca \equiv cb \pmod n$, then prove that $a \equiv b \left(\bmod \frac{n}{d}\right)$ where d = gcd (c,n).
20. Using Euler's theorem, prove that for any integer a, $a^{13} \equiv a \pmod{2730}$.
21. Solve the system of linear congruence equations $7x + 3y \equiv 10 \pmod{16}$, $2x + 5y \equiv 9 \pmod{16}$.
22. Find $\tau(180)$ and $\sigma(180)$.
23. Show that if a, b, c are real, the roots of the equation $\frac{1}{x+a} + \frac{1}{x+b} + \frac{1}{x+c} = \frac{3}{x}$ are real.
24. Prove that in an equation with real coefficients, imaginary roots occur in pairs.
25. Find the roots of the equation $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$.
26. Find the condition that the cubic equation $ax^3 + 3bx^2 + 3cx + d = 0$ has two equal roots.

SECTION– D

Answer **any 2** questions from this Section. **Each** question carries **6** marks. **(2×6=12)**

27. State and prove division algorithm.
28. State and prove Wilson's theorem.
29. a) Find the condition that the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$ may be in geometric progression.

    b) Solve the equation $27x^3 + 42x^2 - 28x - 8 = 0$ whose roots are in geometric progression.
30. Solve the equation $4x^4 + 4x^3 - 7x^2 - 4x - 12 = 0$.

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