Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree (CBCSS – OBE – Supplementary/ Improvement) Examination, April 2025
(2019 to 2023 Admissions)
CORE COURSE IN STATISTICS
2B02STA : Probability Theory and Mathematical Expectation

Time : 3 Hours | Max. Marks : 48

PART – A

Short Answer (Answer **all** questions, **1** mark **each**).

1. Define sample space.
2. What do you mean by mutually exhaustive events ?
3. Define discrete random variable.
4. Define probability density function.
5. State two properties of mathematical expectation.
6. Show that $V(X) = E[X^2] - (E[X])^2$. **(6×1=6)**

PART – B

Short Essay (Answer **any 7** questions, **2** marks **each**).

7. Define the statistical definition of probability.
8. Define the terms :
   a) Mutually exclusive events.
   b) Independent events.
9. Define probability mass function(pmf) and give the properties of pmf.
10. Distinguish between discrete and continuous random variable.
11. A random variable has mean 10 and variance 25. Find for what values of a and b, does the variable Y = aX + b has zero expectation and variance unity.

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12. Two unbiased dice are thrown. Find the expected values of the sum of numbers on them.
13. Define conditional expectation.
14. Calculate the first three raw moments of a continuous random variable X with probability density function given by $f(x) = 2x,\ 0 < x < 1$.
15. Define probability generating function of a random variable X. How will you derive mean of a random variable using its probability generating function ? **(7×2=14)**

PART – C

Essay (Answer **any 4** questions, **4** marks **each**).

16. If A and B are two events such that $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{8}$. Find $P(A/B)$ and $P(A/B^C)$.
17. If A and B are independent events, then show that
    i) A and B^C are independent.
    ii) A^C and B are independent.
18. A random variable X has probability mass function given by $P(X = 1) = 1/2$, $P(X = 2) = 1/3$ and $P(X = 3) = 1/6$. Write down the distribution of X.
19. If X is continuous random variable with probability density function ;
    $$f(x) = \begin{cases} kx, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$
    Find :
    a) k
    b) Write down the probability density function of Y = 3X + 1.
20. In two tosses of a coin, let X be the number of heads. Derive probability distribution of X and hence calculate its skewness.
21. Define characteristic function of a random variable and deduce its properties. **(4×4=16)**

PART – D

Essay (Answer **any 2** questions, **6** marks **each**).

22. i) State Baye's theorem.
    ii) There are two identical boxes containing respectively 4 white and 3 red balls; 3 white and 7 red balls. A box is chosen at random and a ball is drawn from it. It happens to be white. Find the probability that it is from the first box.
23. Let $f(x, y) = \begin{cases} e^{-(x+y)}, & x > 0,\ y > 0 \\ 0, & \text{elsewhere} \end{cases}$
    Find the
    i) Marginal probability density function of X and Y.
    ii) Test whether X and Y are independent.
    iii) Conditional density function of X given Y.
24. Define covariance between two random variables X and Y. Also prove that
    i) cov (aX, bY) = ab cov(X, Y).
    ii) cov (X + a, Y + b) = cov(X, Y).
25. Discuss the procedure for deriving mean and variance of a random variable using its moment generating function. Also prove that
    i) $M_{X+Y}(t) = M_X(t)\, M_Y(t)$, when X and Y are independent.
    ii) $M_{aX+b}(t) = e^{bt} M_X(at)$. **(2×6=12)**

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