Reg. No.: ....

Name : ....

V Semester B.Sc. Hon's (Mathematics) Degree (CBCSS – Regular/ Supplementary/Improvement) Examination, November 2022
(2017 Admission Onwards)
BHM501 : SPECIAL FUNCTIONS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions. **Each** question carries **1** mark.

1. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} n!x^n$.
2. Write down the Gauss's hypergeometric series F(a, b, c, x).
3. Define Gamma function.
4. Define an improper integral.
5. Write down the Rodrigues' formula. **(4×1=4)**

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. Define regular singular point of a second order linear homogenous differential equation and give an example of a differential equation with a regular singular point at x = 0.
7. For the differential equation $(3x + 1)xy'' - (x + 1)y' + 2y = 0$, locate and classify its singular points on the x-axis.
8. Verify $\log(1 + x) = xF(1, 1, 2, -x)$ by examining the series expansions of the functions on the left side.

9. Write down the Gauss's hypergeometric equation and test the nature of its singular points.
10. State the orthogonality property of Legendre polynomials.
11. Prove that $\frac{d}{dx}[J_0(x)] = -J_1(x)$.
12. Investigate the convergence of the integral $\int_0^{\infty} \frac{x^2}{4x^4+25}dx$.
13. Define improper integrals of second kind.
14. Evaluate $\int_0^{\infty} x^6 e^{-2x}dx$, using Gamma function. **(6×2=12)**

SECTION – C

Answer **any 8** questions out of **12** questions. **Each** question carries **4** marks.

15. Find a power series solution of the form $\sum_{n=0}^{\infty} a_n x^n$ to solve the differential equation $y' = 2xy$ and verify your conclusion by solving the equation directly.
16. Find the general solution of $(1 + x^2)y'' + 2xy' - 2y = 0$ in terms of power series in x.
17. Find the indicial equation and its roots for the differential equation $2x^2y'' + x(2x + 1)y' - y = 0$.
18. Find the general solution of the Gauss's hypergeometric equation $x(1 - x)y'' + \left(\frac{3}{2} - 2x\right)y' + 2y = 0$ near the singular point x = 0.
19. Determine the nature of the point $x = \infty$ for the Bessel's equation $x^2y'' + xy' + (x^2 - p^2)y = 0$.
20. Verify that the following by examining the series expansions of the functions on the left sides :

    a) $e^x = \lim_{b\to\infty} F\left(a, b, a, \frac{x}{b}\right)$

    b) $\cos x = \lim_{a\to\infty} F\left(a, a, \frac{1}{2}, \frac{-x^2}{4a^2}\right)$.

21. Prove that $\Gamma(p + 1) = p\Gamma(p)$.
22. Prove that $\frac{d}{dx}\left[x^{-p}J_p(x)\right] = -x^{-p}J_{p+1}(x)$.
23. Prove that any third degree polynomial $p(x) = b_0 + b_1x + b_2x^2 + b_3x^3$ can be expressed as a linear combination of the Legendre polynomials $P_0(x)$, $P_1(x)$, $P_2(x)$ and $P_3(x)$.
24. Test for convergence of the following integrals.

    a) $\int_1^{\infty} \frac{x}{3x^4+5x^2+1}dx$

    b) $\int_2^{\infty} \frac{x^2-1}{\sqrt{x^6+16}}dx$.
25. Prove that

    a) $B(u, v) = B(v, u)$.

    b) $B(u,v) = 2\int_0^{\frac{\pi}{2}} \sin^{2u-1}\theta\cos^{2v-1}\theta\, d\theta$.
26. Evaluate $\int_0^a y^4\sqrt{a^2-y^2}dy$, using Beta and Gamma functions. **(8×4=32)**

SECTION – D

Answer **any 2** questions out of **4** questions. **Each** question carries **6** marks.

27. Find the general solution of $(1 - x^2)y'' - 2xy' + p(p + 1) y = 0$ in terms of power series in x.
28. Find the general solution of the differential equation $(x^2 - 1)y'' + (5x + 4)y' + 4y = 0$ in terms of hypergeometric series near the singular point $x = -1$.
29. State and prove the orthogonality property of Bessel functions.
30. Prove that $B(u, v) = \frac{\Gamma(u)\Gamma(v)}{\Gamma(u+v)}; u, v > 0$ **(2×6=12)**