Reg. No. : ....

Name : ....

V Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/Improvement) Examination, November 2024 (2021 and 2022 Admissions)
5B22 BMH : COMPLEX ANALYSIS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Find the value of $e^{2\pi i}$.
2. Give an example for an entire function.
3. Find $\int_{-i}^{i} \frac{dz}{z}$.
4. Give an example of a divergent sequence.
5. Define pole of order m.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. If $f(z) = 2iz + 6\bar{z}$ then find the value of f(z) at $z = \frac{1}{2} + 4i$.
7. Solve $\sin z = 0$.
8. Find $\int_{8+\pi i}^{8-3\pi i} e^{\frac{z}{2}}\, dz$.
9. Evaluate $\oint_C \frac{dz}{z}$ where C is the unit circle in counter clockwise direction.
10. State Liouville's theorem.

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11. Give an example for conditionally convergent series.
12. Test the convergence of $\sum_{n=0}^{\infty} \frac{z^n}{n^2}$.
13. Evaluate $\oint_C \frac{4-3z}{z^2-z}\, dz$ counter clockwise around simple closed path with 0 is inside and 1 is outside.
14. Find the singularity of $e^{\frac{1}{z}}$.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Prove that $f(z) = \bar{z}$ is not differentiable.
16. If $f(z) = \frac{1}{1-z}$ then find the value of f(z) at z = 1 – i, Ref (z), Img f(z).
17. a) Find the value of $|e^{iy}|$.
    b) Solve $\cos z = 0$.
18. a) Evaluate $\int_C \text{Re}\, z dz$ where C is represented by $z(t) = t + 2it$ $(0 \le t \le 1)$.
    b) Evaluate $\oint_C \frac{e^z}{z-2}\, dz$ for any contour enclosing $z_0 = 2$.
19. Evaluate $\oint_C \frac{e^z}{z^n}\, dz$ (n = 1, 2, 3, ...), C is the unit circle.
20. Find an upper bound for the absolute value of the integral $\int_C z^2\, dz$ where C is the straight line segment from 0 to 1 + i.
21. Show that the series $\sum_{n=0}^{\infty} \frac{(100+75i)^n}{n!}$ is convergent.
22. a) Define a power series.
    b) Test the convergence of the series $\sum_{n=0}^{\infty} z^n$.
23. Find the Maclaurin series expansion of $f(z) = \tan^{-1} z$.

24. Evaluate $\oint_C \frac{dz}{z^3 - z^4}\, dz$ clockwise around the circle $C : |z| = \frac{1}{2}$.
25. Using Residue theorem evaluate $\oint_C \frac{\sin z}{z^4}\, dz$, C is the unit circle in counter clockwise direction.
26. Write the Laurent series expansion of $f(z) = \frac{\sin z}{z^4}$.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. If f(z) is analytic in a domain D and |f(z)| = k = constant in D, then show that f(z) = constant in D.
28. a) Evaluate $\oint_C \frac{\cosh 2z}{\left(z-\frac{1}{2}\right)^4}\, dz$, C is the unit circle.
    b) Evaluate $\oint_C \frac{dz}{z^2}$ where C is the unit circle.
29. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2} (z-3i)^n$.
30. Find all Laurent series expansion of $f(z) = \frac{-2z+3}{z^2-3z+2}$ with center 0 .

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