

K24U 3163

Reg. No. :

Name :

**V Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. –
Supplementary) Examination, November 2024
(2019 and 2020 Admissions)
BHM 503 : ADVANCED DISCRETE MATHEMATICS**

Time : 3 Hours

Max. Marks : 60

PART – AAnswer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Define an Eulerian trail of a graph.
2. State the Ringel-Kotzig Conjecture.
3. State Halls condition.
4. What is meant by derangements ?
5. Define Ferrers graph.

PART – BAnswer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. Draw the de Bruijn digraph $B(3, 2)$.
7. Define the closure of a graph. Explain with example.
8. Give an example of a connected 4-regular graph this is not Hamiltonian factorable.
9. Distinguish maximum matching and maximal matching.
10. Distinguish edge independence number and lower edge independence number.
11. Explain the concept of rook polynomials.

P.T.O.

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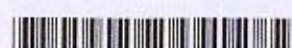
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12. Calculate the number of derangements of 1, 2, 3, 4, 5, 6, 7, 8.
13. Find the generating function for the sequence 0, 1, 2,
14. For each real number n , write the Maclaurin series expansion of $(1 + x)^n$.

PART – CAnswer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Prove that a connected graph G contains an Eulerian trail if and only if exactly two vertices of G have odd degree.
16. i) Explain de Bruijn sequence with an example.
ii) Explain the de Bruijn digraph with an example.
17. If G is a Hamiltonian graph, then prove that $\kappa(G - S) \leq |S|$ for every non-empty proper subset S of $V(G)$.
18. State and prove the Marriage Theorem.
19. Prove that for every non-empty graph G , $\alpha'_o(G) \leq \alpha'(G) \leq 2\alpha'_o(G)$.
20. Prove that for each positive integer k , the complete graph K_{2k} is 1-factorable.
21. Find the number of non-negative integer solutions to the equation $x_1 + x_2 + x_3 + x_4 = 18$ with $x_i \leq 7$ for $1 \leq i \leq 4$.
22. Six married couples are to be seated at a circular table. In how many ways can they arrange themselves so that no wife sits next to her husband ?
23. Let $A = \{1, 2, 3, 4\}$ and $B = \{u, v, w, x, y, z\}$. How many one-to-one functions $f: A \rightarrow B$ satisfy none of the following conditions : $C_1: f(1) = u$ or v , $C_2: f(2) = w$, $C_3: f(3) = w$ or x , $C_4: f(4) = x, y$ or z . Find the number of one-to-one functions $f: A \rightarrow B$ where none of the conditions C_1, C_2, C_3, C_4 is satisfied.
24. In how many ways can a police captain distribute 24 rifle shells to four police officers so that each officer gets at least three shells, but not more than eight ?



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25. Verify that for all $n \in \mathbb{Z}^+$, ${}^{2n}C_n = \sum_{i=0}^n ({}^nC_i)^2$.

26. Using generating functions to determine how many four-element subsets of $S = \{1, 2, 3, \dots, 15\}$ contain no consecutive integers.

PART – DAnswer **any two** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. Let G be a graph of order $n \geq 3$. If $\deg u + \deg v \geq n$ for each pair u, v of nonadjacent vertices of G , then prove that G is Hamiltonian.
28. Prove that a graph G is 2-factorable if and only if G is $2k$ -regular for some positive integer k .
29. State and prove the generalized principle of inclusion and exclusion.
30. Using the exponential generating function, find the number of ways can four of the letters in ENGINE be arranged ?