



M 8869

Reg. No. :

Name :

II Semester B.Sc. Degree (CCSS – 2014 Admn. – Regular)

Examination, May 2015

CORE COURSE IN MATHEMATICS

2B02 MAT : Integral Calculus

Time : 3 Hours

Max. Marks : 48

SECTION – A

All the first 4 questions are **compulsory**. They carry 1 mark each.

1. State the mean value theorem for definite integrals.

2. Evaluate : $\int_0^{\infty} x^3 e^{-x} dx$.3. Fill in the blanks : The equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} + 2z = 0$ represents a surface known as
_____4. Evaluate : $\int_{-1}^1 \int_0^2 (1 - 6x^2y) dx dy$. (4×1=4)

SECTION – B

Answer **any 8** questions from among the questions 5 to 14. They carry 2 marks each.5. Show that if f is continuous on $[a, b]$, $a \neq b$ and if $\int_a^b f(x) dx = 0$ then $f(x) = 0$ at least once in $[a, b]$.6. Define the Riemann sum of a continuous function f defined on the interval $[a, b]$.7. Evaluate : $\int_0^1 \sinh^2 x dx$.

P.T.O.



8. Test for convergence: $\int_1^{\infty} \frac{x dx}{3x^4 + 5x^2 + 1}$.

9. Prove that $\Gamma(n+1) = n \Gamma(n)$, $n > 0$.

10. Find the area between $y = \sec^2 x$ and $y = \sin x$ from 0 to $\pi/4$.

11. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1$, $x = 4$ about the line $y = 1$.

12. Find the length of the curve $y = \frac{4\sqrt{2}}{3} x^{3/2} - 1$, $0 \leq x \leq 1$.

13. Evaluate $\int_0^1 \int_0^x (3 - x - y) dy dx$.

14. Find the area enclosed by the lemniscate $r^2 = 4 \cos 2\theta$. (8x2=16)

SECTION - C

Answer **any 4** questions from among the questions 15 to 20. They carry **4** marks each.

15. Find $\int x \sin^{-1} x dx$.

16. Examine for convergence $\int_0^{\infty} \frac{1 - \cos x}{x^2} dx$.

17. The line segments $x = 1 - y$; $0 \leq y \leq 1$ is revolved about the y-axis to generate the cone. Find its lateral surface area.

18. The region in the first quadrant enclosed by the parabola $y = x^2$, the y-axis and the line $y = 1$ is revolved about the line $x = 3/2$ to generate a solid. Find the volume of the solid.



19. Find the volume of the region enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

20. Evaluate $\int_0^1 \int_0^{1-x} \sqrt{x+y} (y-2x)^2 dy dx$. (4x4=16)

SECTION - D

Answer **any 2** questions from among the questions 21 to 24. They carry **6** marks each.

21. Find the area of the region between the x-axis and the graph of $f(x) = x^3 - x^2 - 2x$; $-1 \leq x \leq 2$.

22. Prove that $B(m, n) = \frac{\Gamma(m) \Gamma(n)}{\Gamma(m+n)}$; $m, n > 0$.

23. Find the area inside the smaller loop of the limaçon $r = 2 \cos \theta + 1$.

24. Find the volume of the upper region D cut from the solid sphere $\rho \leq 1$ by the cone $\phi = \pi/3$. (2x6=12)