



K16U 1221



Reg. No. :

Name :

II Semester B.Sc. Degree (CCSS – Reg./Supple./Improv.)

Examination, May 2016

CORE COURSE IN MATHEMATICS

2B02 MAT : Integral Calculus

(2014 Admn. Onwards)

Time : 3 Hours

Max. Marks : 48

SECTION – A

All the first 4 questions are compulsory. They carry 1 mark each.

1. The set $P = \{0, .2, 6, 1, 1.5, 2\}$ is a partition of $[0, 2]$. Find $\|P\|$.2. If $f(x)$ has the constant value c on $[a, b]$, find $\int_a^b f(x) dx$.

3. Give an example of improper integral of the second kind.

4. Give the equation of the quadric surface known as hyperboloid of one sheet.

(4×1=4)

SECTION – B

Answer any 8 questions from among the questions 5 to 14. They carry 2 marks each.

5. Express the limit of Riemann sums $\lim_{|P| \rightarrow 0} \sum_{k=1}^n (C_k^2 - 3C_k) \Delta x_k$, where P is a partition of $[-7, 5]$.6. Obtain the upper and lower bounds for the value of $\int_0^1 \frac{1}{1+x^2} dx$ given by the max-min inequality.

P.T.O.



7. Express the solution of the following initial value problem in terms of integrals,

$$\frac{dy}{dx} = \sec x, y(2) = 3.$$

8. Examine for convergence: $\int_1^{\infty} \frac{\ln x}{x+a} dx$, where a is a positive constant.

9. Evaluate the integral, $\int_0^{\infty} x^5 e^{-2x} dx$.

10. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

11. Find the volume of the solid generated by revolving the region between the y -axis and the curve $x = \frac{2}{y}$; $1 \leq y \leq 4$ about the y -axis.

12. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$; $1 \leq x \leq 2$ about the x -axis.

13. Sketch the region of integration for the integral $\int_0^{2\sqrt{x}} \int_{x^2}^{2x} (4x+2) dy dx$ and write an equivalent integral with the order of integration reversed.

14. Find the average value of $f(x, y) = x \cos xy$ over the rectangle $R: 0 \leq x \leq \pi, 0 \leq y \leq 1$.

(8x2=16)

SECTION - C

Answer **any 4** questions from among the questions 15 to 20. They carry **4** marks each.

15. Show that the function f defined by $f(x) = \begin{cases} 1, & \text{when } x \text{ is rational} \\ 0, & \text{when } x \text{ is irrational} \end{cases}$ has no Riemann integral over $[0, 1]$.

16. Show that $\int_0^2 x \sqrt[3]{8-x^3} dx = \frac{16\pi}{9\sqrt{3}}$.



17. The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x -axis to generate a solid. Find the volume of the solid.

18. Find the length of the curve $y = \left(\frac{x}{2}\right)^{2/3}$ from $x = 0$ to $x = 2$.

19. Find the average value of $F(x, y, z) = xyz$ over the cube bounded by the coordinate planes and the planes $x = 2$, $y = 2$ and $z = 2$ in the first octant.

20. Evaluate $\int_0^4 \int_{x=y/2}^{(y/2)+1} \frac{2x-y}{2} dx dy$

by applying the transformation $u = \frac{2x-y}{2}$, $v = y/2$ and integrating over an appropriate region in the uv -plane.

(4x4=16)

SECTION - D

Answer **any 2** questions from among the questions 21 to 24. They carry **6** marks each.

21. Find $\int \sin^2 x \cos^3 x dx$.

22. Prove that $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$.

23. Find the length of the astroid $x = \cos^3 t$, $y = \sin^3 t$; $0 \leq t \leq 2\pi$.

24. Find the volume of the region D enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

(2x6=12)