



Reg. No. : .....

Name : .....



K18U 0641

Second Semester B.Sc. Degree (CBCSS – Regular) Examination, May 2018  
(2017 Admission)

CORE COURSE IN MATHEMATICS  
2B02MAT : Integral Calculus

Time : 3 Hours

Max. Marks : 48

### SECTION – A

Answer **all** questions from **1** to **4**. Each question carries **1** mark.

1. State the Mean Value Theorem for Definite Integrals.
2. State Fundamental Theorem of Calculus that describes how to evaluate definite integrals.
3. Define Beta Function.
4. Let  $f$  be smooth on  $[a, b]$ . Define the length of the curve  $y = f(x)$  from  $a$  to  $b$ . (4×1=4)

### SECTION – B

Answer **any eight** questions among the questions **5** to **14**. Each question carries **2** marks.

5. Graph the integrand and use area to evaluate  $\int_{-2}^1 |x| dx$ .
6. Using the definitions of Hyperbolic sine and Hyperbolic cosine prove that  $\int \sinh u du = \cosh u + C$ .
7. Using reduction formula evaluate  $\int \tan^3 x dx$ .

P.T.O.



8. Evaluate  $\int_0^{\infty} x^3 e^{-x^2} dx$ .

9. Prove that  $B(m, n) = B(n, m)$  where  $B$  denotes the Beta Function.

10. The region between the curve  $y = \sqrt{x}$ ,  $0 \leq x \leq 4$  and the x-axis is revolved about the x-axis to generate a solid. Find its volume.

11. Using integration find the length of the curve  $x = a \cos t$ ,  $y = a \sin t$ ,  $0 \leq t \leq 2\pi$ .

12. Find the area inside one loop of the lemniscate  $r^2 = 4 \sin 2\theta$ .

13. Find the average value  $f(x, y) = x \cos xy$  over the rectangle  $R: 0 \leq x \leq \pi$ ,  $0 \leq y \leq 1$

14. Evaluate  $\int_0^1 \int_0^1 \int_0^1 (x^2 + y^2 + z^2) dz dy dx$ . (8×2=16)

### SECTION – C

Answer **any four** questions among the questions **15 to 20**. Each question carries 4 marks.

15. Using reduction formula evaluate  $\int \sin^2 2\theta \cos^3 2\theta d\theta$ .

16. Prove that  $B(p, q) = \int_0^1 \frac{x^{p-1} + x^{q-1}}{(1+x)^{p+q}} dx$ .

17. The region bounded by the curve  $y = x^2 + 1$  and the line  $y = -x + 3$  is revolved about the x-axis to generate a solid. Find the volume of the solid using the Washer method.

18. Find the length of the cardioid  $r = 1 - \cos \theta$ .

19. Evaluate  $\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y dx dy$ , by reversing the order of integration.

20. Find the volume of the upper region  $D$  cut from the solid sphere

$\rho \leq 1$  by the cone  $\phi = \frac{\pi}{3}$ .

(4×4=16)



### SECTION – D

Answer **any two** questions among the questions **21 to 24**. Each question carries 6 marks.

21. Using the integral for the area as a limit of Riemann sums, find the area of the region between the parabola  $y = x^2$  and the x-axis on the interval  $[0, b]$ .

22. Define Gamma function. State and prove factorial property of the Gamma function. Express  $n\Gamma(n)$  in terms of  $n!$ .

23. Find the area of the surface generated by revolving the curve  $y = x^3$ ,  $0 \leq x \leq \frac{1}{2}$  about the x-axis.

24. A thin plate covers the triangular region bounded by the x-axis and the lines  $x = 1$  and  $y = 2x$  in the first quadrant. The plate's density at the point  $(x, y)$  is  $\delta(x, y) = 6x + 6y + 6$ . Find plate's mass, first moments, center of mass, moments of inertia and radii of gyration about the coordinate axes. (2×6=12)