



K20U 0310

Reg. No. : .....

Name : .....

**II Semester B.Sc. Degree (CBCSS – Supplementary/Improvement)**

**Examination, April 2020**

**CORE COURSE IN MATHEMATICS**

**2B02 MAT : Integral Calculus**

**(2017-2018 Admissions)**

Time : 3 Hours

Max. Marks : 48

**SECTION – A**

Answer **all** questions from **1 to 4**. **Each** question carries **1** mark.

1. State Fundamental Theorem of Calculus (Part 2).
2. State the Mean Value Theorem for Definite Integrals.
3. Define Gamma function.
4. Let  $f$  be smooth on  $[a, b]$ . Define the length of the curve  $y = f(x)$  from  $a$  to  $b$ .

**(4x1=4)**

**SECTION – B**

Answer **any eight** questions among the questions **5 to 14**. **Each** question carries **2** marks.

5. Find the area of the region between the curve  $y = 4 - x^2$ ,  $0 \leq x \leq 3$  and the  $x$ -axis.
6. Using the definitions of Hyperbolic sine and Hyperbolic cosine prove that  $\int \sinh u \, du = \cosh u + C$ .
7. Derive the reduction formula for  $\int (\ln x)^n \, dx$ .
8. Evaluate  $\int_0^1 \left( \ln \frac{1}{y} \right)^{n-1} dy$  using the definition of Gamma Function.
9. Prove that  $B(m, n) = B(n, m)$  where  $B$  denotes the Beta Function.
10. Using integration find the length of the curve  $x = a \cos t$ ,  $y = a \sin t$ ,  $0 \leq t \leq 2\pi$ .
11. Find the volume of the solid generated by revolving the region between the  $y$ -axis and the curve  $x = \frac{2}{y}$ ,  $1 \leq y \leq 4$  about  $y$ -axis.

P.T.O.



12. The standard parametrization of the circle of radius 1 centered at the point (0, 1) in the xy-plane is  $x = \cos t$ ,  $y = 1 + \sin t$ ,  $0 \leq t \leq 2\pi$ . Using this parametrization find the area of the surface swept out by revolving the circle about the x-axis.
13. Evaluate  $\int_0^1 \int_0^1 \int_0^1 (x^2 + y^2 + z^2) dz dy dx$ .
14. Find the average value of  $f(x, y) = \sin(x + y)$  over the rectangle  $0 \leq x \leq \pi$ ,  $0 \leq y \leq \pi$ . (8×2=16)

## SECTION – C

Answer **any four** questions among the questions 15 to 20. Each question carries 4 marks.

15. Prove that  $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$ ,  $-\infty < x < \infty$ .
16. Prove that  $B(p, q) = \int_0^1 \frac{x^{p-1} + x^{q-1}}{(1+x)^{p+q}} dx$ .
17. The region bounded by the curve  $y = x^2 + 1$  and the line  $y = -x + 3$  is revolved about the x-axis to generate a solid. Find the volume of the solid using the washer method.
18. Find the length of the cardioid  $r = a(1 - \cos \theta)$ .
19. Evaluate  $\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y dx dy$ , by reversing the order of integration.
20. A solid of constant density  $\delta = 1$  occupies the region D cut from the solid sphere  $\rho \leq 1$  by the cone  $\phi = \frac{\pi}{3}$ . Find the solid's moment of inertia about the z-axis, using triple integrals in spherical coordinates. (4×4=16)

## SECTION – D

Answer **any two** questions among the questions 21 to 24. Each question carries 6 marks.

21. Using the integral for the area as a limit of Riemann sums, find the area of the region between the parabola  $y = x^2$  and the x-axis on the interval  $[0, b]$ .
22. Prove that  $B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ .
23. Find the area inside the smaller loop of the limaçon  $r = 2 \cos \theta + 1$ .
24. A thin plate covers the triangular region bounded by the x-axis and the lines  $x = 1$  and  $y = 2x$  in the first quadrant. The plate's density at the point  $(x, y)$  is  $\delta(x, y) = 6x + 6y + 6$ . Find plate's mass, first moments, center of mass, moments of inertia and radii of gyration about the coordinate axes. (2×6=12)