



K19U 0265

Reg. No. :

Name :

II Semester B.Sc. Degree (CBCSS – Reg./Supple./Improv.)

Examination, April 2019

(2017 Admission Onwards)

Core Course in Mathematics

2B02MAT : INTEGRAL CALCULUS

Time : 3 Hours

Max. Marks : 48

SECTION – A

Answer **all** questions from 1 to 4. Each question carries 1 mark.

1. Find $\Gamma(1)$, where Γ is the Gamma function.
2. State the formula for the length of a smooth curve $x = g(y)$, $c \leq y \leq d$.
3. State Fubini's Theorem (First form).
4. Define the average value of a function F over a region D in space. (4×1=4)

SECTION – B

Answer **any eight** questions among the questions 5 to 14. Each question carries 2 marks.

5. Find the area of the region between the curve $y = 4 - x^2$, $0 \leq x \leq 3$ and the x-axis.
6. Derive the reduction formula for $\int (\ln x)^n dx$.
7. Evaluate $\int_0^1 \left(\ln \frac{1}{y} \right)^{n-1} dy$ using the definition of Gamma function.

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8. Using the definition of Beta function, evaluate $\int_0^1 x^4(1-x)^3 dx$.
9. Find the volume of the solid generated by revolving the region between the y-axis and the curve $x = \frac{2}{y}$, $1 \leq y \leq 4$, about y-axis.
10. The standard parametrization of the circle of radius 1 centered at the point (0, 1) in the xy-plane is $x = \cos t$, $y = 1 + \sin t$, $0 \leq t \leq 2\pi$. Using this parametrization find the area of the surface swept out by revolving the circle about the x-axis.
11. Using integration find the area enclosed by the polar curve $r = a$, $0 \leq \theta \leq 2\pi$, a is a constant.
12. Evaluate $\int_0^3 \int_0^2 (4-y^2) dy dx$.
13. Using triple integral find the volume of the cube bounded by the coordinate planes and the planes $x = 2$, $y = 2$, $z = 2$ in the first octant.
14. Find the average value of $f(x, y) = \sin(x+y)$ over the rectangle $0 \leq x \leq \pi$, $0 \leq y \leq \pi$. (8×2=16)

SECTION - C

Answer **any four** questions among the questions 15 to 20. Each question carries 4 marks.

15. State and prove the Mean value theorem for definite integrals.
16. Prove that $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$, $-\infty < x < \infty$.
17. Prove that $\int_0^{\frac{\pi}{2}} \sin^{2m-1} \theta \cos^{2n-1} \theta d\theta = \frac{\Gamma(m)\Gamma(n)}{2\Gamma(m+n)}$ where Γ is the Gamma function.



18. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$, $1 \leq x \leq 2$ about x-axis.
19. Find the perimeter of the cardioid $r = a(1 - \cos\theta)$.
20. A solid of constant density $\delta = 1$ occupies the region D cut from the solid sphere $\rho \leq 1$ by the cone $\phi = \frac{\pi}{3}$. Find the solid's moment of inertia about the z-axis, using triple integrals in spherical coordinates. (4×4=16)

SECTION - D

Answer **any two** questions among the questions 21 to 24. Each question carries 6 marks.

21. State and prove the Fundamental Theorem of Calculus (Part 2).
22. Prove that $B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.
23. Find the area inside the smaller loop of the limaçon $r = 2\cos\theta + 1$.
24. Find the centroid of the solid enclosed by the cylinder $x^2 + y^2 = 4$, bounded above by the paraboloid $z = x^2 + y^2$ and below by the xy-plane (given $\delta = 1$). (2×6=12)